Introduction to Reliability Engineering

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Lecture 20

System Reliability Modelling (Contd.)

Hello everyone. So, we will now move on to our next topic of discussion that is Markov analysis. In previous lectures, we discussed system relative modeling using relative block diagram, where we have discussed series system, parallel system, k out of m system, etcetera. So, there we assume that we have a system which is composed of components and component reliability if you know we are able to know the system reliability.

Similar modeling can also be done using the Markov systems. In Markov analysis, we are trying to do the analysis based on the system states. So, there are various possible combinations of system states depending on the component states and generally from one state to another state there will be a transition, there is a possibility that a system can change state from one state to another state.

So, considering that if we are able to know the various states and we know the probability of various states, then based on the state probabilities we can determine the reliability. Reliability is nothing but the probability (we are the) of the states where the system is working and unreliability is the probability that of the states where the system is not working.

So, this concept we will try to explore because the state-based system like standby system or where the one changes or changing the system configuration and because of that change need to be considered, in those cases relative block diagram approach may not be sufficient enough to solve the problems. So, we will try to use them Markov analysis for those purpose.

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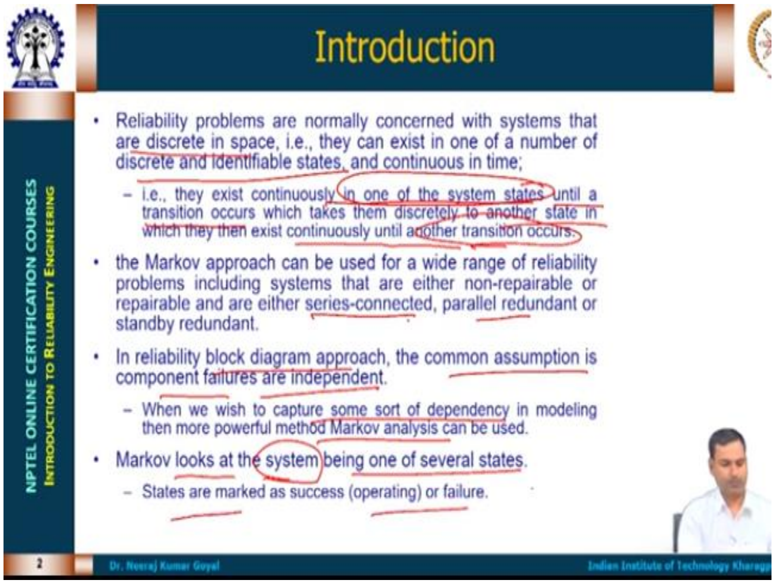

So, if we look at here reliability problems are normally concerned with systems that are discrete in space. So, system can exist in one of a number of discrete and identifiable states. So, some of these states will be failure states and some of these states will be successes states. As we discussed earlier, as I told earlier that system states are discrete which is a combination where it is telling whether some systems are some components of the system are working or not working and based on the combination we determine whether the system is working going to work in this case or not.

Here, these system states are, (())(3:05) continuously in one of the system state. So, if system we are considering, the system will be in any one of these states, all these states are mutually exclusive, the system can exist in one of these state until a transition occurs. So, when something happens, some failure happens or some system component state changes, depending on the system state is also changing and system moves to another state which is, so, this is the component of the system can remain in that state continuously until there is another transition happening that means, either another failure happens or some failed component is repaired and it is starting to working again.

The Markov approach can be used for a variety wide range of reliability problems including systems that are non-repairable or repairable. There can be series connected, parallel connected or can be standby. Generally, series connected parallel connect redundant systems we are easily solved using the reliability block diagram approach. A Markov when we are going to use there are some limitations and there is some, it is little bit complex than the reliability block diagram approach.

So, we will try to use reliability block diagram approach where this reliability block diagram approach is not feasible, there we try to go further Markov approach. In reliability block diagram approach, the common assumption is that component failures are independent. However, we want to capture some sort of dependency. Sometimes what can happen one component failure can put more load on the system, so in that case what will happen? The dependency is no longer independency is no longer there.

So, (there) these kinds of dependencies if we are able to represent, then we can use the Markov analysis. Markov analysis tries to put the system into various states and all these states of system will be some of the states will be marked as success some will be marked as a failure.

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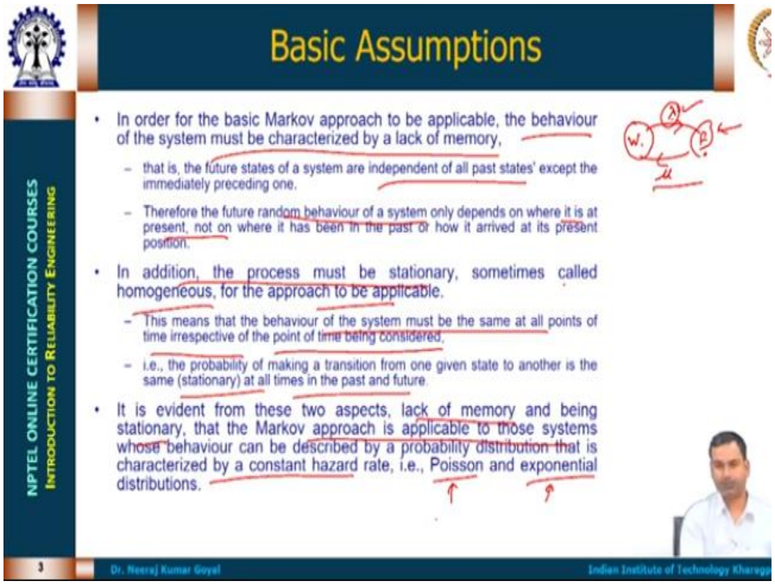

General assumptions when we are doing Markov analysis, so Markov analysis cannot be solved for all types of problem, but Markov analysis is only applicable where the behavior of the system is characterized by a lack of memory. Like same, memory less property we discussed in case of exponential distribution that means, irrespective of the age of the component, the failure rate remains same, failure rate do not change or the conditional probability of failure per unit time does not change with the age.

Similar case is here, here when we use Markov diagram, we will be having the state diagram. So, here it is possible that our system is in good state, it gets failed, then it get repaired and come back to the good state again. So, there is a possibility that there may be many such transition or the system can accumulate age.

Here, the in Markov diagram when we consider, if we consider lack of memory we consider that transition probabilities or the state probabilities which we are getting they will not depend on the age, they will depend on the current that where the system was in last state. So, failure state of the systems are independent of all past states. So, whether it has been failed two times or three times it does not make much difference failure and repair, failure and repair let us say we have one state, here it is a component is working., then, let us say we talk about refrigerator, so refrigerator is working here now, some failure happens it will move to the failed state, then what will happen we can call the technician and technician will repair this. So, it will fail by the rate of failure rate and it will repair by the repair rate.

Now, what happens the failure rate which you are denoting here or the probability of failure per unit time which we are denoting here that is considered to be time independent. So, that is why if there are let us say 4-5 such transition, in that case also we will assume that the age will not impact. Like, if fridge is 10 year old, then this will this should not change the failure rate and repairs should be seen. So, this phenomena, whenever is possible we are trying to use Markov modeling.

So, the failure rate lambda will depend only on the previous state that is the W or this mu will depend on this where it is starting from that is F, whether the system is in this state or not that will only determine. So, the future random behavior of a system depends only where it is at present and it does not depend on where it has been in past, whether it has been in various states of which states it has traveled in past, that will not change the probability.

So, this property Markovian property has to be true, then only we can use the Markov diagram. If that property is not true, then we have we cannot use the Markov in this form but there are other forms of Markov like semi Markov or Markov chain etcetera, they may be used for that purpose. The process should also be stationary that is also called homogeneous, that means the behavior of system must be same at all points of time irrespective of the time being considered.

So, it should not depend on age. The probability of making transition from one given state to another state is going to be same, this lambda is not going to be a function of time, then only we can use this continuous Markov chain Markov analysis.

So, from these two aspects we can we show that lack of memory and being stationary, then only we can use a Markov approach for these systems. So, generally this is applicable only when our failure if we are talking about the failure process, then the failure process has to be Poisson. And if you are talking about (ex) if it is failure rate, then we are seeing it is the exponential distribution,, constant failure rate. In both the cases, in Poisson and exponential both the cases as we discussed the failure rate is constant, so, this property has to be followed.

As we discussed as I mentioned that even if this property is not followed it does not mean that we cannot solve it, but for that we have to use a more complex Markov approach, this approach as we are going to discuss may not be applicable.

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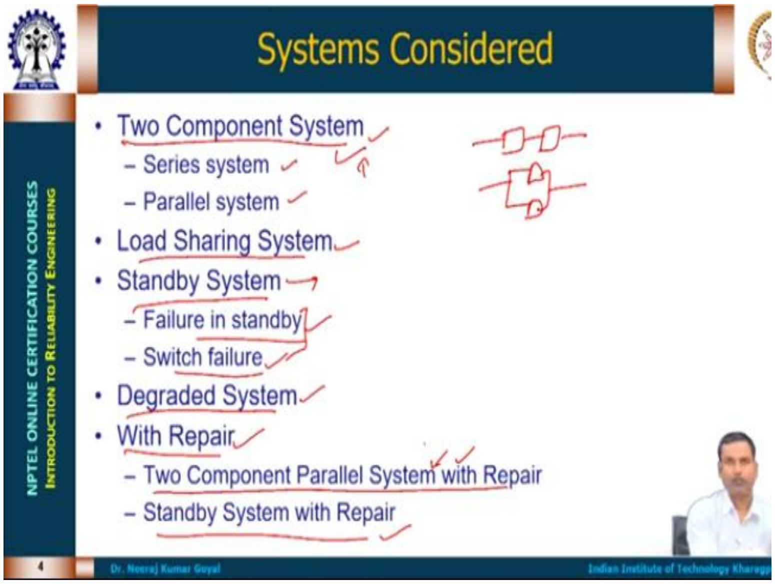

The systems we will be discussing for Markov analysis is, first we will discuss a two component system. Two component systems can be in two ways either they can be in series or they can be in parallel. So, we can discuss both the cases that if we have that two component, if we are able to model two component system using Markov, then we will get the different states probabilities and based on the different state probabilities depending on whether our system is configured a series of parallel, we will be able to know what is the reliability for series system or parallel system.

Then we will discuss load sharing system, load sharing systems are the one where load is shared. So, like we have motors, we have generators or some other plant or the equipments. These equipments, when they are all working together, then the load which is coming on this equipment is shared load or less load. But if one of the equipment fails, then that load transferred to the working equipment. So, because of that what happens these devices starts working on the higher load and we know that when stress is high for any system, then failure probability of failure rate will also be high.

So, that system which is remaining and keep on working, those components will observe a little higher failure rate than they will usually observe when the load is shared. So, these kinds of systems we can model using the Markov diagram. We will also discuss standby systems, standby systems are actually like our generators or we have standby motors also we have standby pumps also, so, they are still there they are in that system, but they are not continuously operating, they are in the standby mode. Whenever some failure happens, then the standby unit will come into the picture and that will replace the main unit and it will start working in that case.

Standby unit we will consider failures. There are two possibilities we will consider, failure in standby that means, there is a possibility that the unit which we have kept in standby, when we need to plug in that unit for our operation at that time that unit is found as the failed unit, that is no longer working because in the standby mode itself it has failed, so, that (constitution) condition will also can also be taken care into the Markov analysis.

There is another possibility similar like we discussed for motors or there can be generators etcetera. So, whenever there is let us say if we talk about the general generator, then if the power failure occurs, then generator will automatically switch and that will be coming taking care of the power requirement. But if the switch itself fails, then what will happen, then also we will not be able to get the power supply. So, switch failure if we consider, then how the standby system will behave all these can be considered.

There is a possibility of degraded system consideration. Degraded system is almost similar to load sharing system sometimes like we have let us say some system let us say we have bearings which is being used in various parts or let us say if you use any like motorcycle car etcetera. So, what can happen that after a certain period of time these systems or their components can get degraded. During degradation cannot completely failed, but their performance has degraded as well as their failure rate has become higher, because in degraded state when we are going to use those items, the chances of failures will be high. So, degraded systems also we can model and we can consider different failure rates when they are degraded and not degraded.

Then with repair also systems can be considered. Whenever we consider repair, then modeling becomes a little tough but we will discuss about it how to take care of it. And once we take repair into consideration, then we will consider a simple case that is the two component system here. If we consider this is, first we will consider without repair, then we will consider that if they are if this is we have the repair facility and the failed equipment can be repaired, then how much will be the reliability we will be getting.

Similarly, we will also consider the standby system. Here, we will consider without repair, then we will also consider an example if that that if repair is considered, then how the reliability can be calculated.

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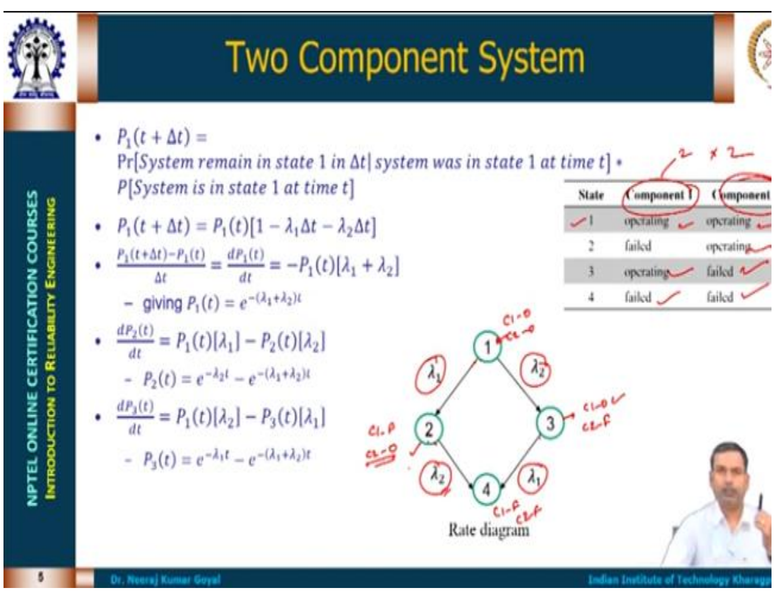

Let us take an example of two component system. In two component system, we have two components here; component 1, component 2. Now, there are various possibilities here emerging out of it, because as here we are considering that each component can be in two state, either it can be in operating state or it can be failed state.

$$P_1(t+\Delta t) = P_1(t)[1 - \lambda_1 \Delta t - \lambda_2 \Delta t]$$
$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = \frac{dP_1(t)}{dt} = -P_1(t)[\lambda_1 + \lambda_2]$$
$$\text{giving } P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$
$$\frac{dP_2(t)}{dt} = P_1(t)[\lambda_1] - P_2(t)[\lambda_2]$$
$$P_2(t) = e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$
$$\frac{dP_3(t)}{dt} = P_1(t)[\lambda_2] - P_3(t)[\lambda_1]$$
$$P_3(t) = e^{-\lambda_1 t} - e^{-(\lambda_1+\lambda_2)t}$$

So, we have two states for component 2 and we have two states for component 1, so, all together we have 2 into 2, 4 states. These are the 4 states which are mentioned here that is, both components are operating component 1 is failed component 2 is operating, component 1 is operating component 2 is failed or both are failed.

Now, so, we have 4 states here, first state is both are operating. So, here we have state number 1 where we have component 1 is also operating and component 2 is also operating. Now, (there is) there are two possibilities here from this state either component 1 fails or component 2 fails. So, if component 1 can fail with the transition rate of lambda 1 because failure rate for component 1 is lambda 1.

So, from this state there is a transition rate of lambda 1 by which we can move to the system state number 2. In the system state number 2, component 1 is failed state but component 2 is working from this state there is another transition possible that is that is due to the failure rate of second component. So, if second component fails in that case component 1 is (opera) continues to continue to operate but (com) component 2 that is failed.

Now, from these two state if we see, only component 2 is operating. So, it says component 2 is operating, then failure it will be lambda 2, so, with that this state if you see component 1 is failed and component 2 is also failed. From this state also same state can be achieved by because component 1 is operating failure rate of component 1 is lambda 1. So, from component this state to this state where component 1 and 2 both are in failed state (the fail) the rate will be lambda 1. So, this is how we are able to make this transition diagram or rate diagram. In rate diagram, what we do we try to explore all possibilities here all states are mentioned here.

So, here all four states of this system are mentioned here and as we see that in each transition only one failure is considered or only one change is considered in one transition two change will not be considered that is also another Markov assumption. So, in one change, only one change will happen. So, here lambda 1 is changing of component 1 is failing a component 2 is failing, then here because component 2 is operating, so component 2 fails here and here component 1 is operating, so, component 1 fails here. So, with this diagram once we know we are able to let us move forward, let us see how we can analyze this.

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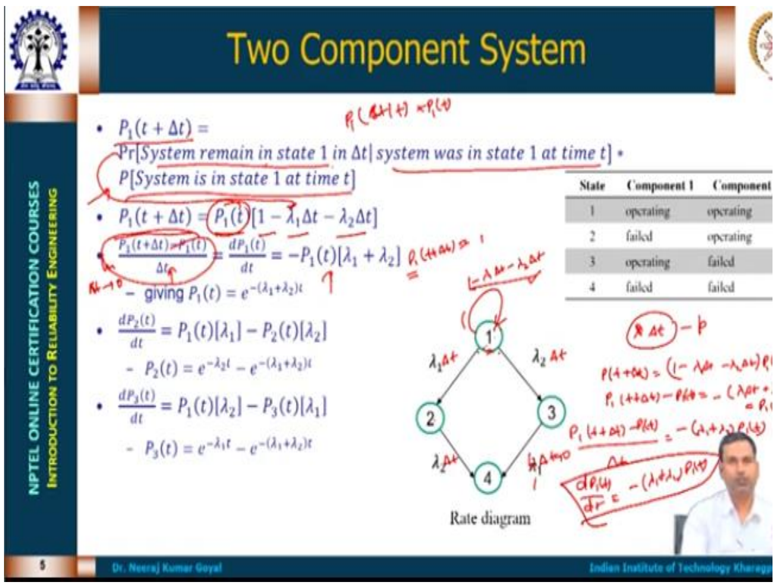

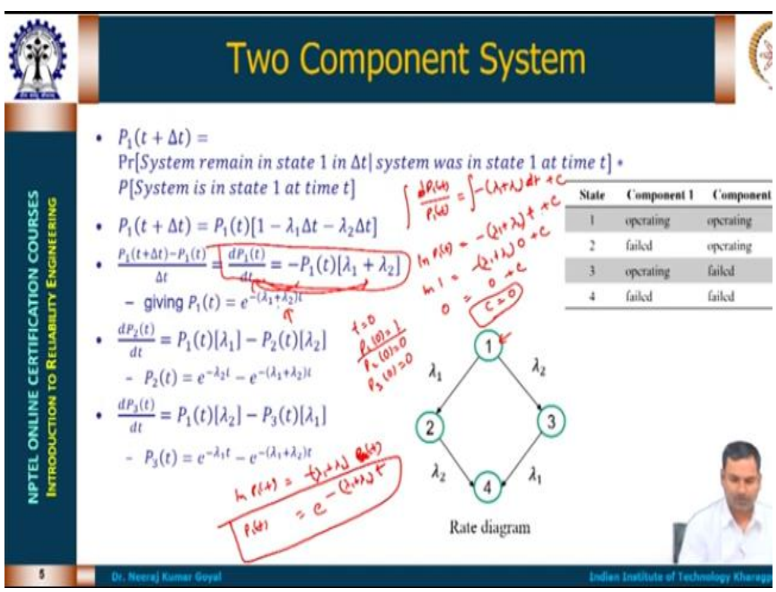

Now, let us see this, we want to develop the equations here. So, here to develop the equations first let us see, let us assume that we want to know the P1 t plus delta t. What is P1 t plus delta t? P1 means probability of that system is in state 1 at time t plus delta t. Now, the system can be in state 1 at time t plus delta t, that is only possible if system was in state 1 at time t and it moves to the same state that is state 1 that means, it does not fail neither it moves to state 2 nor it moves to state 3.

Now, these rates we have mentioned in terms of rates. So, we know lambda is the rate lambda is the conditional probability of failure per unit time. So, if we multiply lambda with delta t, this becomes probability of failure in delta t time, small time delta t the probability of failure or the chain probability that system will move from system state 1 to state 2 is lambda 1 delta t. Similarly, this we can write probabilities lambda 2 delta t, this is lambda 1 delta t, this is lambda 2 delta t.

Now, the probability for state 1 to move the probability that system can be out from state 1 that is, lambda 1 delta t and lambda 2 delta t. So, what is the probability that system will be in state remaining in state 1? That means, it does not move out that is 1 minus lambda 1 delta t minus lambda 2 delta t. So, the system will remain in state 1 in time delta t. The probability is 1 minus lambda 1 delta t minus lambda 2 delta t because lambda 1 delta t is the probability it can move the state 2 and lambda 2 delta t is the probability that it can move to state 3.

So, the probability that it will remain in the same state is 1 minus lambda 1 delta t lambda 2 delta t given that it was already state 1. So, the probability of transition from state 1 to state 1 becomes this. So, here probability that the system is in state 1 at time t plus delta t, this probability is only possible that this is the probability that system remains in state 1 only that means, this probability, but under one condition, the condition is that at the time t system should be in state 1 because here no other condition is happening like system one cannot system cannot reach the state 1 from state 2 there is no transition possible.

Similarly, from 3 there is no transition possible. Similarly, from 4 there is no transition possible to state 1. So, we have the 0 possibility, we have only one possibility that system can be in state 1 at time t plus delta t that is, that system was in state at was in state 1 at time t and it remains in same state. So, that is the probability that system remains in state 1 in time in time delta t given that, system was in a state 1 at time t multiply with the probability that system is in, we know that Pt plus delta t is p delta t we can say given t multiply by P1 t.

So, how much is P1 t plus t delta t given t that is we already know from here that is 1 minus lambda 1 delta t minus lambda 2 delta t. So, P1 t plus delta t is nothing but P1 t into 1 minus lambda 1 delta t minus lambda 2 delta t this probability is 1 minus lambda 1 delta t minus lambda 2 delta t multiplied by probability that system is in state t state 1 at time t that is P1 t.

So, here as we discuss P t plus delta t is nothing but probability that system does not transit from here that is 1 minus lambda 1 delta t minus lambda 2 delta t multiply by probability that system should be in state 1, that is P1 t. So, this if we solve this we can take P1 t plus delta t and from here this will become all multiplied by P1 delta t, so, (my) P1 t will be coming here 1 into P1 t as P1 t, so, P1 t if we take left this will become minus and minus lambda 1 delta t minus of minus minus plus lambda 2 delta t into P1 t.

Now, as we see here the same thing we have written here, delta t if we take common from both, then this will become P1 t plus delta t minus P1 t divided by delta t will be equal to minus of lambda 1 plus lambda 2 into P1 t. Same thing we have written here. So, here, then we put limit delta t tending to 0, then this delta p limit delta t tending to 0 when we put, then this become dP1 t over dt. So, dP1 t over dt is equal to minus of lambda 1 plus lambda 2 into P1 t. This becomes a differential equation simple very simple differential equation which we can solve very easily here, I will show you how this comes because just to remind you how…

So, we have got this equation. Now we want to solve this. So, what how we can solve? We can say this equation dP1 t if we take P1 to this side and dt to this side. So, this will become dP1 t over P1 t will be equal to minus lambda 1 plus lambda 2 into dt. Now, if you take integration on both side and if you take plus C, then this is dP1 t over P1 t so, this will give the ln of P1 t this is equal to minus lambda 1 plus lambda 2 both are time independent, so, we can take them outside and integration of dt will give you t plus C.

Now, if we put the condition here that is, we know at time t equal to 0 system F at the start of when we started the system at the time t equal to 0 the system was in state 1. So, the P1 0 is equal to 1. Similarly, but P2 0 will be equal to 0 and P3 0 will be equal to 0 because at the time t equal to 0 system was in state 1. So, probability of system being in state 1 P1 0 is equal to 1.

Now the same thing if we put, then ln of P1 0 that is ln of 1 will be equal to minus lambda 1 plus lambda 2 t 0 is equal to plus C, lambda ln value of 1 is equal to 0 and this is also 0 this is C. So, C will be equal to 0.

When C equal to 0 we put, then this equation will become ln of P1 t is equal to minus lambda 1 plus lambda 2 into P1 t, into t. Now here if we solve this, then P1 t will be equal to if you take this here this will become exponential e to the power minus lambda 1 plus lambda 2 into t the same, I have written here that P1 t is equal to e to the power minus lambda 1 plus lambda 2 into t. So, we have solved this equation and we could after solving this differential equation we could get this P1 t.

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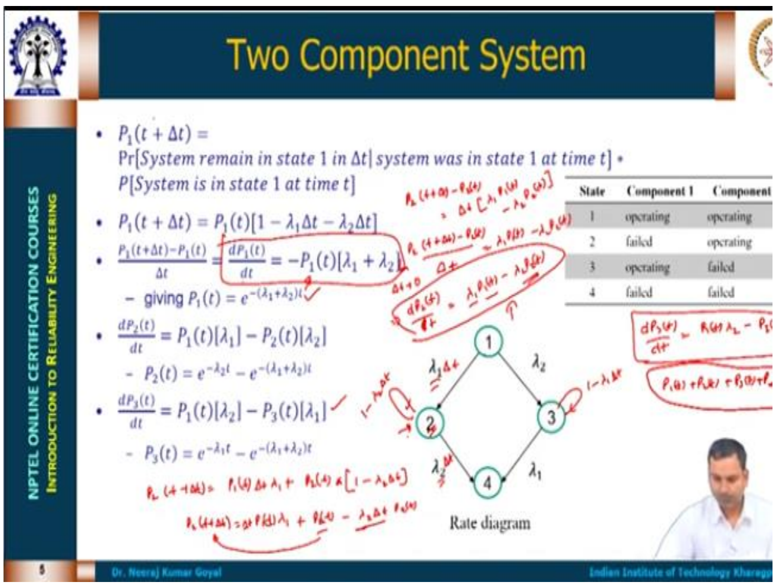

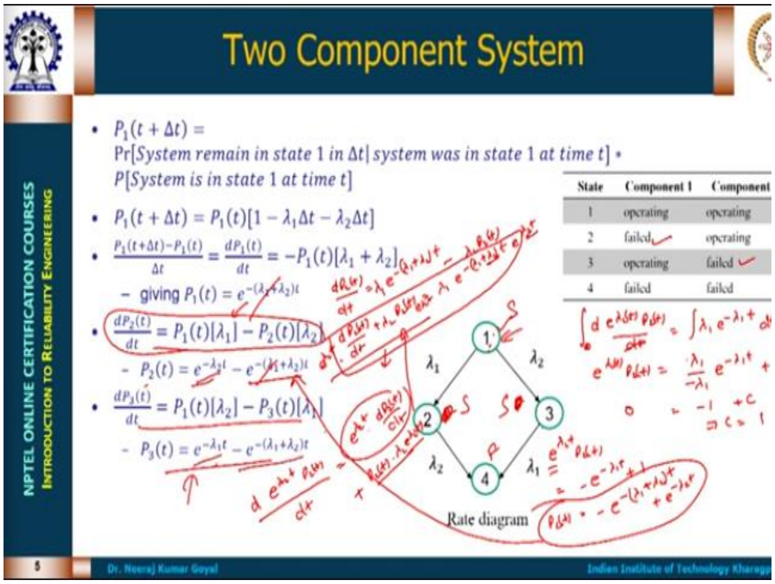

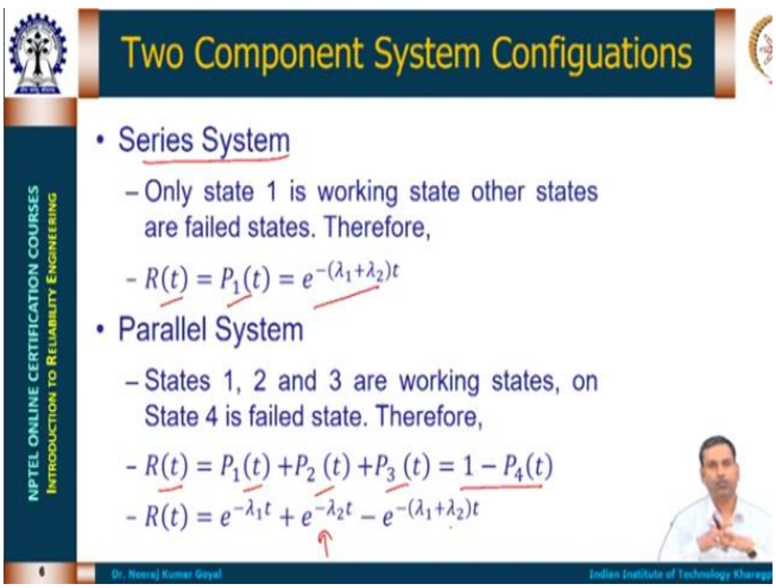

Similarly, we can get other probabilities; now for state 2 if we write the equation, then for state 2 what will be the probability that it will stay in the same state is 1 minus outgoing probabilities. So, it can go outgoing probabilities lambda 2 delta t, so, the probability that system is in state 2 and it remains in state 2 that means it does not go out from the system state, so, that will be 1 minus lambda 2 delta t. Similarly, if we see this, this will be 1 minus lambda 1 delta t.

Now, if we can make the same equation here, then for P2 t, so, P2 t plus delta t will be equal to now P2 t there are two possibilities that system is in state 1 at time t that means P1 t and system transit from state 1 to state 2 the transition probabilities lambda 1 delta t delta t lambda 1. Now, the second possibility is that system is in state 2 and it remains in state 2 that is probability that system is in state 2 that is P2 t multiplied by probability that is, system is remains in state 2 that is 1 minus lambda 2 delta t.

Now, if we solve this will become P2 t plus delta t equal to P1 t into lambda 1 plus P2 t I will keep delta t minus lambda 2 delta t t2 t. Now, if you see we take this towards that this P2 t we take this side so this will become P2 t plus delta t minus P2 t that will be equal to delta t delta t also we can take common, then it will become lambda 1 P1 t minus lambda 2 P2 t. So, we can write it as P2 t plus delta t minus P2 t divided by delta t equal to lambda 1 t1 t minus lambda 2 P2 t. So, this again when we put limit delta t tending to 0 this will be dP2 t over dt will be equal to lambda 1 t1 t minus lambda 2 P2 t.

We have used these equations to derive, but the to derive these equations, we may not need to do always the same thing. If you look closely at this figure, then we can easily find out like if you look at the first equation here. So, first equation how can we get the how much is the change in P1 t? The change in P1 t will be positive if something is coming that if some transition is there which is incoming to the state that will be positive and if there is some transition which is outgoing to the state that will be negative.

So, P1 t can change, the change is only negative because only outgoing possibilities there and there are two possibilities lambda 1 delta d from state 1. So, from state 1 P1 t lambda 1 and second rate is lambda 2. So, P1 t lambda 1 plus lambda 2, because both are outgoing, so, both are negative here.

Similarly, if we look at secondary state, we have incoming probability rate is lambda 1 from P1. So, lambda 1 from P1 and outgoing is lambda 2, so, from P2 we have the outgoing minus lambda 2. So, that is how we can make the transition probabilities can be directly written and we can directly get these equations. Like for 3, if I want to write the same equation, then dP3 over dt will be equal to of incoming is lambda 2s from state 1. So, P1 t lambda 2 and outgoing is lambda 1 from state 3. So, P3 t into lambda 1.

When we are solving these equations like same equation is written here, when we solve these equations, there is another equation which is always going to true that is P1 t plus P2 t plus P3 t plus P4 t is always going to be 1 the system has to be in one of these a state system cannot be outside these states, so, the state probabilities always sum to 1.

Now, let us just try to solve this that if you are going to solve this equation P2 t, I am erasing this because it is already written here. So, we will go ahead with this, this is the equation. Now, in this equation, if we replace P1 t from here, then this will become dP2 t over dt is equal to e to the power minus lambda 1 plus lambda 2 into t into lambda 1 minus lambda 2 P2 t. So, here this becomes a differential equation dP2 t over dt plus lambda 2 P2 t is equal to lambda 1 e to the power minus lambda 1 plus lambda 2 into t. Here again we can use the same concept as we discussed earlier, we can use the differential equation to solve this.

So, if you use differential equation solving, then what we can do we can multiply both sides with e to the power lambda 2t e to the power lambda 2t e to the power lambda 2t e to the power lambda 2t. Then we solve this because now this term left hand side term we know e to the power lambda 2t P2 t, if you take differentiation of this with respect to t, what we will get? We will get by parts we will get e to the power lambda 2t into dP2 t over dt plus if we can take the (sec) first part for differentiation, then P2 t will remain as it is multiply by lambda 2 e to the power lambda 2t.

So, this is nothing but this same part lambda 2 e to the power lambda 2t P2 t and this part is same as this part. So, this left hand side is nothing but differentiation of e to the power lambda 2t P2 t over dt. So, our equation has become differentiation of e to the power lambda 2t into P2 t over dt is equal to all.

Now, here e to the power lambda 2 t when we multiply, then this will become lambda 1 e to the power minus lambda 1t because this is minus lambda 1 minus lambda 2t and lambda 2t lambda 2t will get cancelled. Now, this equation is if you solve into, so, if we solve this, then we take dt here and, then we can integrate we can integrate from 0 to t maybe. So, when we integrate on both side, then this integration will become e to the power lambda 2t lambda 2t positive and multiply by P2 t will be equal to, this is integration of lambda 1 e to the power minus lambda 1t.

So, lambda 1 will be constant and when we differentiate this, this will we will be getting minus 1 upon lambda 1 e to the power minus lambda 1 t plus C. Now, here if we solve this let us say at t equal to 0 P2 t 0. So, when we put t equal to 0 this will become 0 and, then we put t equal to 0 this will be equal to minus 1 plus C. So, C will be equal to 1 here. When we put C equal to 1 here, then e to the power lambda 2t P2t will be equal to minus e to the power minus lambda 1 t plus 1.

Now, lambda 2 t which we have here we can divide it here. So, P2t will be equal to minus e to the power minus lambda 1 minus again because divide this will become minus, so that cumulatively we can say minus minus plus plus e to the power minus lambda 2t e to the power minus lambda 2t minus e to the power minus lambda 1 plus lambda 2t, same answer we have got.

So, similar way if we can solve for P3 t also and we will get this same equation if you see nothing differences there only lambda rather than lambda 2 it becomes lambda 1. So, this will become lambda 2 in place that lambda 1 and lambda 1 replaced by lambda 2 lambda 1 lambda 2 lambda 2 lambda you will get the same answer. So, we are able to get the P1 t P2 t and P3 t.

Now, there can be two cases here that is series system if you talk about series system, in series system this only is the success state because if anyone is failed if component 2 is failed or component 1 is failed. So, this is failed state this has failed state this is failed state but this is only success state. So, for reliability evaluation, the reliability will be the probability that system is in state 1 at time t that this P1 t. So, Rt becomes P1 t that is e to the power minus lambda 1 plus lambda 2t which is same as we discussed in during reliability block diagram approach.

But if we talk about parallel system, in case of parallel system one failure is allowed only one device working is required. So, this becomes success this becomes success this becomes success. So, we have P1 P2 and P3 three states are success states over reliability by P1 t plus P2 t plus P3 t. So, reliability here becomes P1 t plus P2 t plus P3 t or we can say 1 minus P4 t.

Now, P1 t is e to the power minus lambda 1 plus lambda 2t and P2 t e to the power minus lambda 2t this is minus this, this and this will get cancelled and this will be e to the power minus lambda 1t plus e to the power minus lambda 2t minus e to the power minus lambda 1 plus lambda 2t and same thing comes out here, which is same as what we have got during the parallel (comp) configuration. So, this discussion we will continue in further lectures for other system state system configurations. Thank you.