Introduction to Reliability Engineering
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Lecture 27
Failure Data Analysis: Non- Parametric Approach (Contd.)

Hello, everyone, welcome back to our discussion on failure data analysis. So, we have been discussing that if we have the failure data, how can we analyze the data without using any model and we are able to evaluate the reliability and other parameters. So, regarding that last time we discussed that there can be three approaches using which when we when we sort the data into the order and we find out the data order and then we try to find out the unreliability and then let us go forward and let us see that how this how this works.

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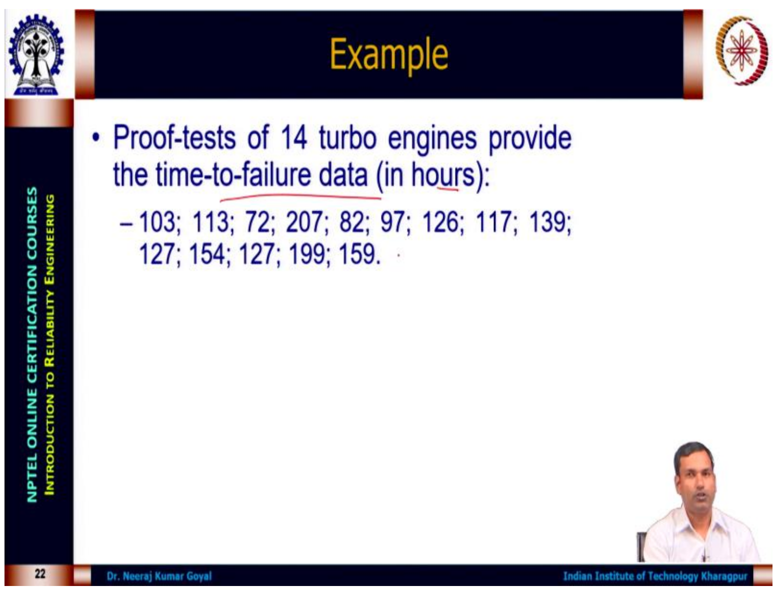

So, let us take an example that a proof of test of 14 turbo engines provide time to failure data. So, that means, in hours so, we have tested 14 turbo engines and we are able to find this time to failure data. Now, this data as we see this is the turbo engine wise, turbo engine 1 failed at 103; 2 failed 113, but this is not ordered.

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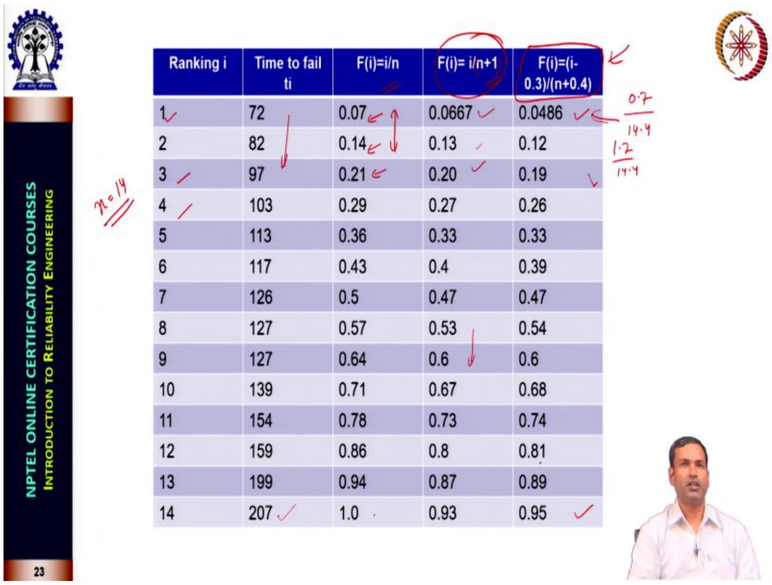

Ranking i	Time to fail ti	F(i)=i/n	F(i)= i/n+1	F(i)=(i-0.3)/(n+0.4)
1	72	0.07	0.0667	0.0486
2	82	0.14	0.13	0.12
3	97	0.21	0.20	0.19
4	103	0.29	0.27	0.26
5	113	0.36	0.33	0.33
6	117	0.43	0.4	0.39
7	126	0.5	0.47	0.47
8	127	0.57	0.53	0.54
9	127	0.64	0.6	0.6
10	139	0.71	0.67	0.68
11	154	0.78	0.73	0.74
12	159	0.86	0.8	0.81
13	199	0.94	0.87	0.89
14	207	1.0	0.93	0.95

So, first what we need to do, we need to order the data this 14 failure data which we have there, I have already put it in Excel and in the Excel we have done the order in the increasing order. So, first failure occurred at 72, second occurred 82. Like that, we have the total 14 failures. Now, as we discussed, we discuss the three approaches equal rank method, whenever we are using equal rank method, then failure probability or unreliability is given us i divided by n, how much is i here, i is 1.

So 1 is here, i is 2 here, i is 3 here, so we have the i values here, and how much is n? n here is 14, 14 devices were put on test. So when i put it divided by n, that means, 1 divided by 14 which gives me 0.07, 2 divided by 14 is 0.14, 3 divided by 14 0.21. So, this is how I am able to get the whole data. And this gives me the unreliability when I am using the equal rank method, if I am using the mean rank method, in mean rank method, as we discussed, this is i divided by n plus 1. So, that means, whatever is the value of i here, 1 that I will divide by the 15 here rather than 14, I will divide by 15.

So, when I divide by 15, I get these values, all the values I have got like this, I have done Excel sheet also, I will show you maybe, little later how to do this in Excel sheet. Then we can also get using the median rank, median rank whenever I am using the same thing I can get it like 0.0486, that is i minus 0.3 divided by 0.4. So, i minus 0.3 means 1 minus 0.3 is 0.7 and n plus 0.4 means 14.4, that will give me this value.

Similarly, when I am using other that is 1.7 divided by 14.4, 2.7 divided by 14.4 like that, we will whenever we use we will be getting this Fi values. So, as we discussed generally the most popular formula is this and also many times this is also used, but I would prefer that if you can use this medium ranking formula for the purpose.

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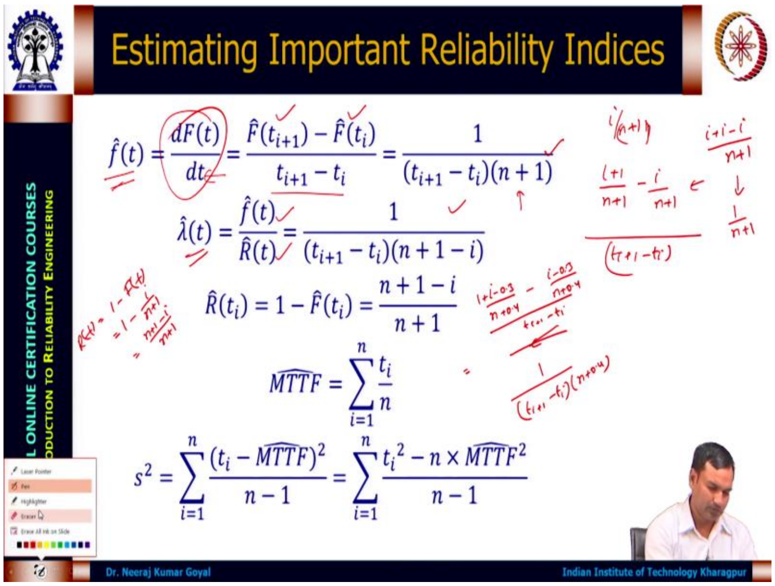

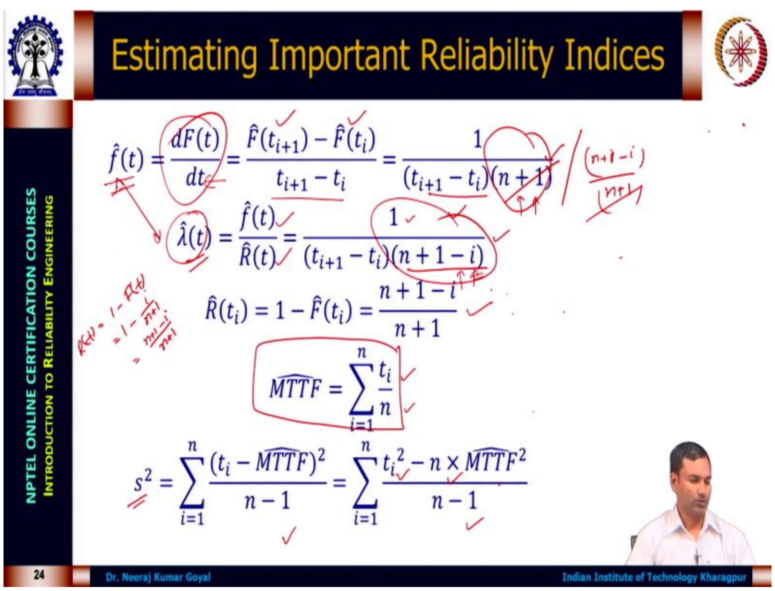

Now, once you get the f t value, that is failure probability, how can we get the PDF value? So PDF value as we know PDF value is the differentiation of f t value. That means if I take Fti plus 1, and if I take Fti. And if I take the difference of the 2 that will give me the difference in ft dFt.

And divided by the difference in time, difference in time is ti plus 1 minus ti. So this gives me the difference in failure probability per unit time.

$$\hat{f}(t) = \frac{dF(t)}{dt} = \frac{\hat{F}(t_{i+1}) - \hat{F}(t_i)}{t_{i+1} - t_i} = \frac{1}{(t_{i+1} - t_i)(n+1)}$$

$$\hat{\lambda}(t) = \frac{\hat{f}(t)}{\hat{R}(t)} = \frac{1}{(t_{i+1} - t_i)(n+1-i)}$$

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = \frac{n+1-i}{n+1}$$

$$\widehat{MTTF} = \sum_{i=1}^{n} \frac{t_i}{n}$$

$$s^2 = \sum_{i=1}^{n} \frac{\left(t_i - \widehat{\text{MTTF}}^{\,2}\right)^n}{n-1} = \sum_{i=1}^{n} \frac{t_i^{\,2} - n \times \widehat{\text{MTTF}}^{\,2}}{n-1}$$

Now, as we know that if we use the i upon n plus 1 formula, if I use this formula, then what will happen Fti plus 1 is i plus 1 divided by n plus 1 and Fti minus 1 is i divided by n plus 1, and whole divided by ti plus 1 minus ti. Now, if you see here this portion if you solve then this will become i plus 1 minus i divided by n plus 1 and this will become 1 divided by n plus 1. So, effectively what I get 1 divided by n plus 1 divided by ti plus 1 minus ti that is 1 divided by ti plus 1 minus ti divided by n plus 1. So, this becomes my PDF value.

So, PDF value is nothing but 1 divided by the difference in time divided by the n plus 1, same thing if you are using any other formula, let us if you use median ranking, for median ranking, you would be using i minus 0.3 divided by n plus 0.4 minus. So, this will be i plus 1 minus 0.3 divided by n plus 0.4 and other is i minus 0.3 divided by n plus 0.4 that is the, and this if you solve the only change would become n divided by ti plus 1 minus ti.

So, if you solve this then what will happen this will become ti plus 1 minus ti will remain same and here if you take n plus 0.4 as common this will become n plus 0.4 and this subtraction would be same. So, i plus 1 minus i minus 0.3 will get canceled by minus 0.3 and only 1 will be remaining. So, if you use median ranking rather than n plus 1 this will become n plus 0.4, the rest of the things could remain same. If you use equal ranking then this will be only n this will not be n plus 1 or neither n plus 0.4 but, so, depending on the method you are using you can use this formula.

Next is lambda t. So, lambda t we know is ft upon Rt, ft upon Rt. So, ft is here already and how much is Rt? What is Rt here? Rt is 1 minus ft and for n plus 1 formula 1 minus i divided by n plus 1 so, this will become n plus 1 minus i divided by n plus 1. So, this value I am removing certain erasing this so, that you can follow it up. So, our Rt value is 1 minus ft that is n plus 1 minus i divided by n plus 1 and ft is this. So, this divided by n plus 1 minus i divided by n plus 1.

If I do this what will happen n plus 1 and n plus 1 will get canceled because this will become n plus 1 upon n plus 1 n plus 1 minus i. So, this n plus 1 will get replaced by n plus 1 minus i. So, what I will have is this is 1 upon ti plus 1 minus ti divided by n plus 1 minus i, n plus 1 get canceled by n plus 1 and n plus 1, 1 minus i will replace this n plus 1.

So, here what is happened in lambda t the change happened is here if we let us say if we use another formula that is if we use the median ranking formula then that can also be used the fact this we are discussing with reference to the mean ranking, but same thing is applicable to any formula which you use.

So, lambda t if we look at it, what does it mean that here in ft when we were considering it was the one of number of failures out of number of units which were there at the start that is n, but here it is number of failures that is 1 out of a number of units which is working at the start of the interval that means, the failure units are not counted. So, i units which are failed at the start of this interval have been removed.

So, failure rate and the difference in failure rate and ft as we discussed in define earlier in our lectures, that ft gives you the number of failures, number of probability of failure per unit time this is the time and this is the probability of failure out of all population, but lambda t gives me that failure probability out of only the surviving population that means not from n plus 1, but from surviving population that is n plus 1 minus i, the failure unit have been removed out of that what is the per unit time failure probability that is given by the lambda t, Rti, is as we have already seen that is n plus 1 minus i upon n plus 1.

For complete data we are able to calculate MTTF also which is nothing but the average value time to failure. So, if you sum up all the time and divide by n that will give the MTTF. And if we want to calculate the this sample variance we can calculate the sample variance as Ti minus MTTF 4 squared divided by n minus 1 and this can also be calculated ti square minus n into f MTTF squared divided by n minus 1.

So, this gives us the formulas which are required to calculate the important quantities of our interests. We want to know failures per unit time that is ft, we want to know failures out of surviving population per unit time that is our lambda t. We want to know reliability, we want to know unreliability, we want to know MTTF we want to know the standard deviation everything we are able to calculate using these formulas.

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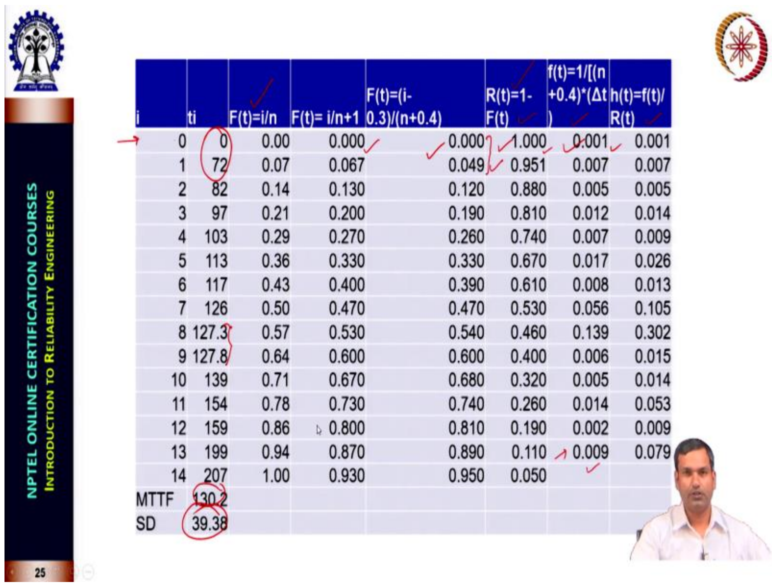

i	ti	F(t)=i/n	F(t)= i/n+1	F(t)=(i-0.3)/(n+0.4)	R(t)=1-F(t)	f(t)=1/[(n+0.4)*(Δt)	h(t)=f(t)/R(t)
0	0	0.00	0.000	0.000	1.000	0.001	0.001
1	72	0.07	0.067	0.049	0.951	0.007	0.007
2	82	0.14	0.130	0.120	0.880	0.005	0.005
3	97	0.21	0.200	0.190	0.810	0.012	0.014
4	103	0.29	0.270	0.260	0.740	0.007	0.009
5	113	0.36	0.330	0.330	0.670	0.017	0.026
6	117	0.43	0.400	0.390	0.610	0.008	0.013
7	126	0.50	0.470	0.470	0.530	0.056	0.105
8	127.3	0.57	0.530	0.540	0.460	0.139	0.302
9	127.8	0.64	0.600	0.600	0.400	0.006	0.015
10	139	0.71	0.670	0.680	0.320	0.005	0.014
11	154	0.78	0.730	0.740	0.260	0.014	0.053
12	159	0.86	0.800	0.810	0.190	0.002	0.009
13	199	0.94	0.870	0.890	0.110	0.009	0.079
14	207	1.00	0.930	0.950	0.050		
MTTF	130.2						
SD	39.38						

Now, if we look at the another example. So, same example which we have taken earlier, we took a 0, here we have added 1 more row here for 0, because at 0 all the system is starting to work and if you see here, then here we have done the correction little bit correction here that at 127 we had 2 failures, since we had 2 failures, so that difference of time will become 0. So, we tried because this is continuous variable. So that there may be a little variation in that, so here the 127 failure, which we had here is actually given at more correct value that is 127.3 and 127.8.

So, we may approximately give like a near about value like 127.25 and 127.75 here or 126.75 and 127.25 here, so that will be on average giving you the same value. So, accordingly we can choose and we can put it up. So ti value we keep it here and Fti as we discussed i upon n, so i is 0 here. So, this will these values we have already calculated this also we have already calculated. Now let us see what will be the reliability value, reliability is 1 minus unreliability.

So, 1 minus of this value will give you the unreliability, this reliability and unreliability values which are calculated is generally given at the start of the interval. So, this unreliability values which are getting we are getting here and this then 0.951. So, I think that is using this ft value. So, this median ranking if you are using them from based on the median ranking Rt values will be these and how much will be the ft values small ft means, change in capital FT divided by time.

So, either what I can do I can take the difference here. So, that is 0.049 divided by time, time is 72. So, 0.049 divided by 72 will give me this value. So, here as you see, this is the difference till next interval, so, this will be 1 less value, because last interval will not be counted this is for the applicable for the interval that means from 0 to 1 failure, or 1 to 2 failure, second, third second to third failure.

These values are calculated and what is the HT or lambda t or hazard rate, that is nothing but whatever ft value we have calculated, if we divide this by reliability we will get this and how much is MTTF here, MTTF is nothing but the average time to ti value here and standard deviation can also be directly calculated using the formula or we have the formula already given in the Excel sheet. So, let me show you how can we do this in the Excel sheet.

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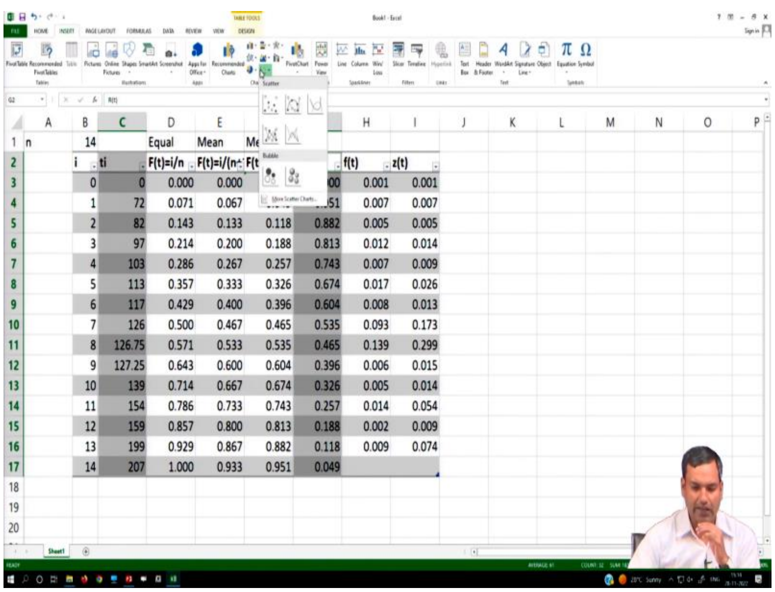
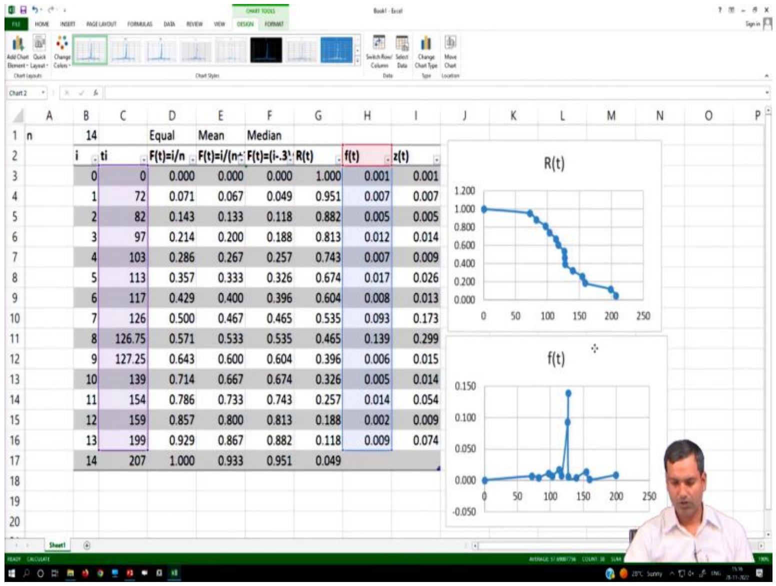
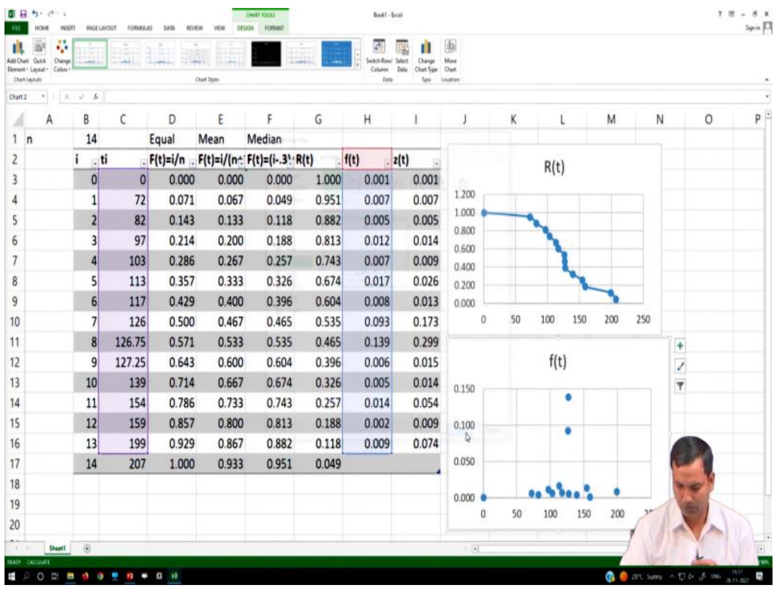




	A	B	C	D	E	F	G	H	I
1	n	14		Equal	Mean	Median			
2		i	ti	F(t)=i/n	F(t)=i/(n+	F(t)=(i-.3	R(t)	f(t)	z(t)
3		0	0	0.000	0.000	0.000	1.000	0.001	0.001
4		1	72	0.071	0.067	0.049	0.951	0.007	0.007
5		2	82	0.143	0.133	0.118	0.882	0.005	0.005
6		3	97	0.214	0.200	0.188	0.813	0.012	0.014
7		4	103	0.286	0.267	0.257	0.743	0.007	0.009
8		5	113	0.357	0.333	0.326	0.674	0.017	0.026
9		6	117	0.429	0.400	0.396	0.604	0.008	0.013
10		7	126	0.500	0.467	0.465	0.535	0.093	0.173
11		8	126.75	0.571	0.533	0.535	0.465	0.139	0.299
12		9	127.25	0.643	0.600	0.604	0.396	0.006	0.015
13		10	139	0.714	0.667	0.674	0.326	0.005	0.014
14		11	154	0.786	0.733	0.743	0.257	0.014	0.054
15		12	159	0.857	0.800	0.813	0.188	0.002	0.009
16		13	199	0.929	0.867	0.882	0.118	0.009	0.074
17		14	207	1.000	0.933	0.951	0.049		



	A	B	C	D	E	F	G	H	I
1	n	14		Equal	Mean	Median			
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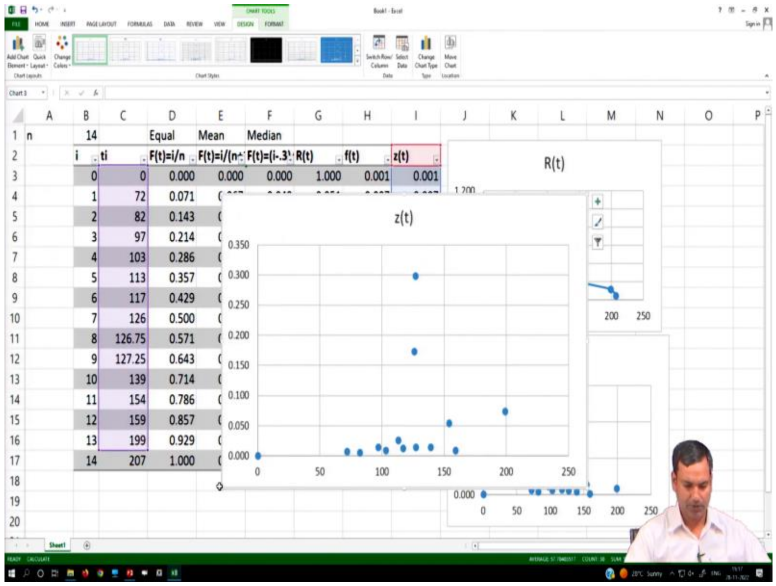
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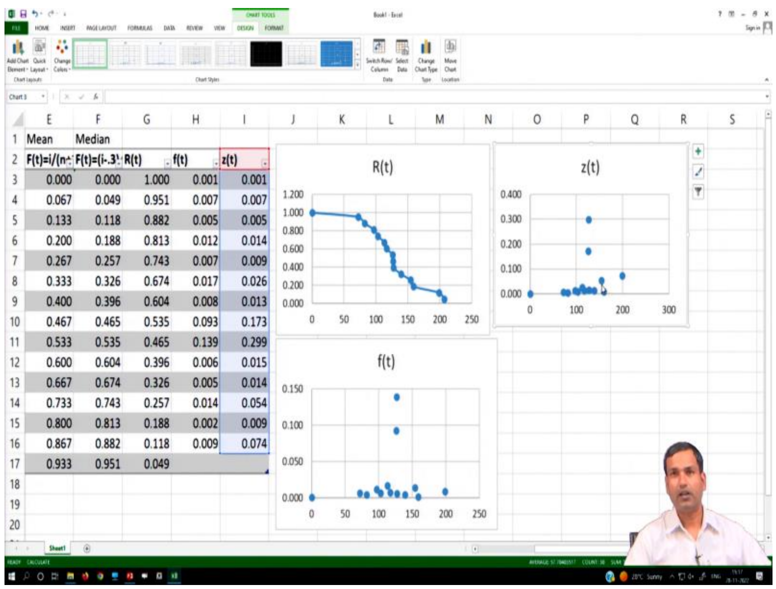
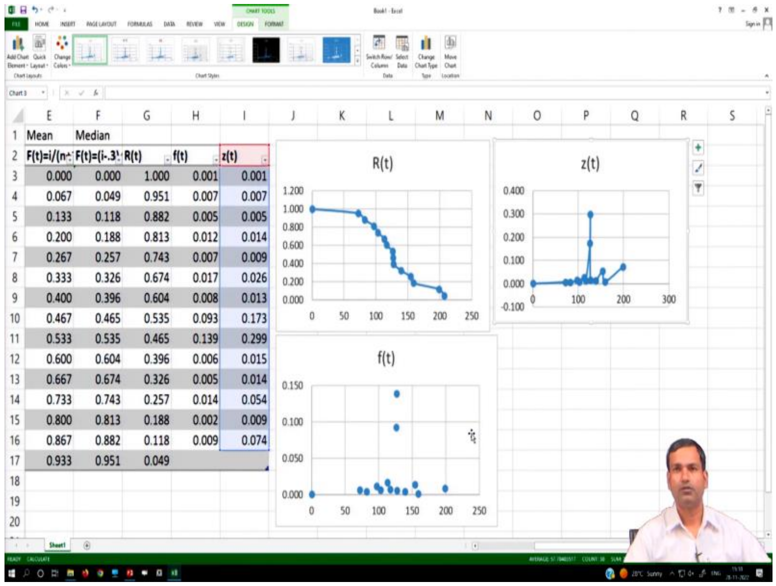
Equal
Mean
Median
i
ti
F(t)=i/n
F(t)=i/(n+
F(t)=(i-.3
R(t)
f(t)
z(t)
0 0 0.000 0.000 0.000 1.000 0.001 0.001
1 72 0.071
2 82 0.143
3 97 0.214
4 103 0.286
5 113 0.357
6 117 0.429
7 126 0.500
8 126.75 0.571
9 127.25 0.643
10 139 0.714
11 154 0.786
12 159 0.857
13 199 0.929
14 207 1.000
z(t)
R(t)
Sheet1


Mean
Median
F(t)=i/(n+
F(t)=(i-.3
R(t)
f(t)
z(t)
0.000 0.000 1.000 0.001 0.001
0.067 0.049 0.951 0.007 0.007
0.133 0.118 0.882 0.005 0.005
0.200 0.188 0.813 0.012 0.014
0.267 0.257 0.743 0.007 0.009
0.333 0.326 0.674 0.017 0.026
0.400 0.396 0.604 0.008 0.013
0.467 0.465 0.535 0.093 0.173
0.533 0.535 0.465 0.139 0.299
0.600 0.604 0.396 0.006 0.015
0.667 0.674 0.326 0.005 0.014
0.733 0.743 0.257 0.014 0.054
0.800 0.813 0.188 0.002 0.009
0.867 0.882 0.118 0.009 0.074
0.933 0.951 0.049
R(t)
z(t)
f(t)
Sheet1


Mean
Median
F(t)=i/(n+
F(t)=(i-.3
R(t)
f(t)
z(t)
0.000 0.000 1.000 0.001 0.001
0.067 0.049 0.951 0.007 0.007
0.133 0.118 0.882 0.005 0.005
0.200 0.188 0.813 0.012 0.014
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0.333 0.326 0.674 0.017 0.026
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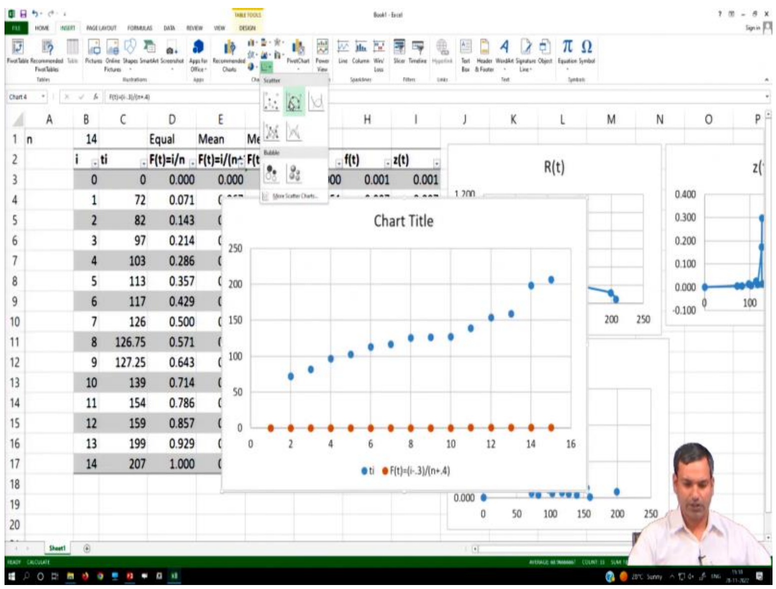
0.667 0.674 0.326 0.005 0.014
0.733 0.743 0.257 0.014 0.054
0.800 0.813 0.188 0.002 0.009
0.867 0.882 0.118 0.009 0.074
0.933 0.951 0.049
R(t)
z(t)
f(t)


n 14
i
ti
Equal
F(t)=i/n
Mean
F(t)=i/(n+
0 0 0.000 0.000
1 72 0.071
2 82 0.143
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7 126 0.500
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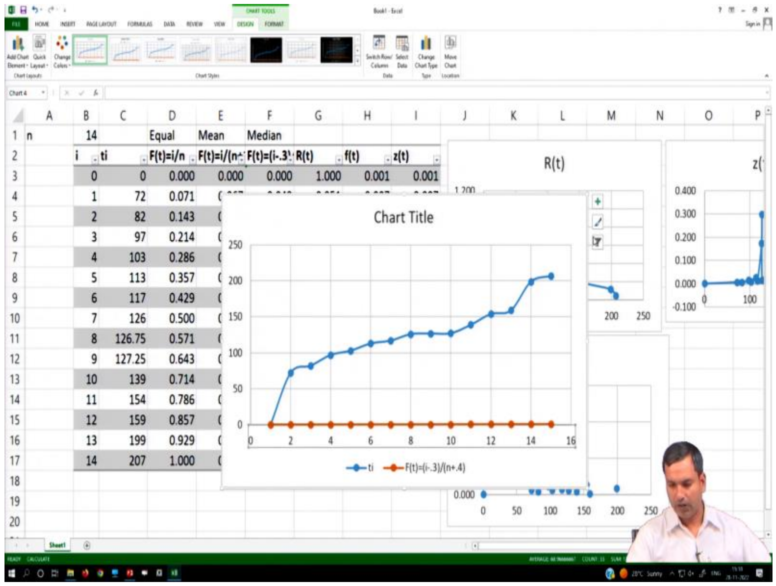
10 139 0.714
11 154 0.786
12 159 0.857
13 199 0.929
14 207 1.000
Chart Title
ti
F(t)=(i-.3)/(n+.4)
R(t)


n 14 Equal Mean Median
i ti F(t)=i/n F(t)=i/(n+ F(t)=(i-.3) R(t) f(t) z(t)
0 0 0.000 0.000 0.000 1.000 0.001 0.001
1 72 0.071
2 82 0.143
3 97 0.214
4 103 0.286
5 113 0.357
6 117 0.429
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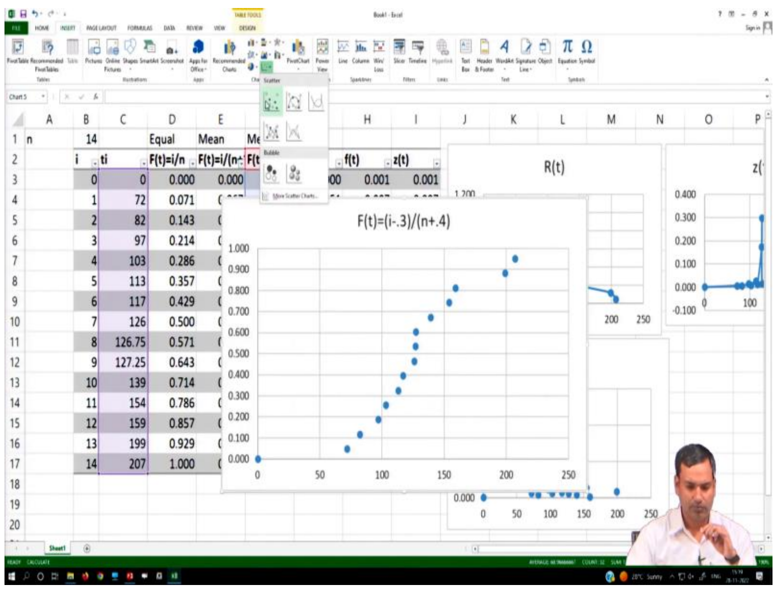
n 14 Equal Mean
i ti F(t)=i/n F(t)=i/(n+ f(t) z(t)
0 0 0.000 0.000 0.001 0.001
1 72 0.071
2 82 0.143
3 97 0.214
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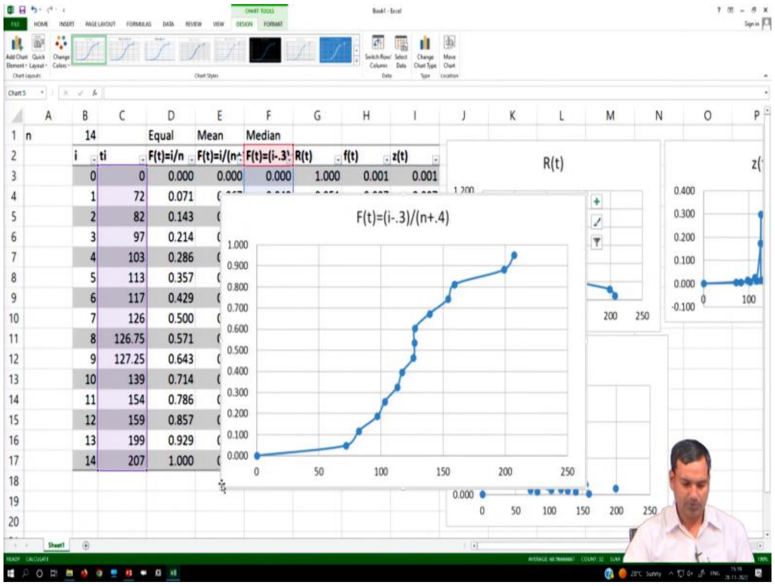
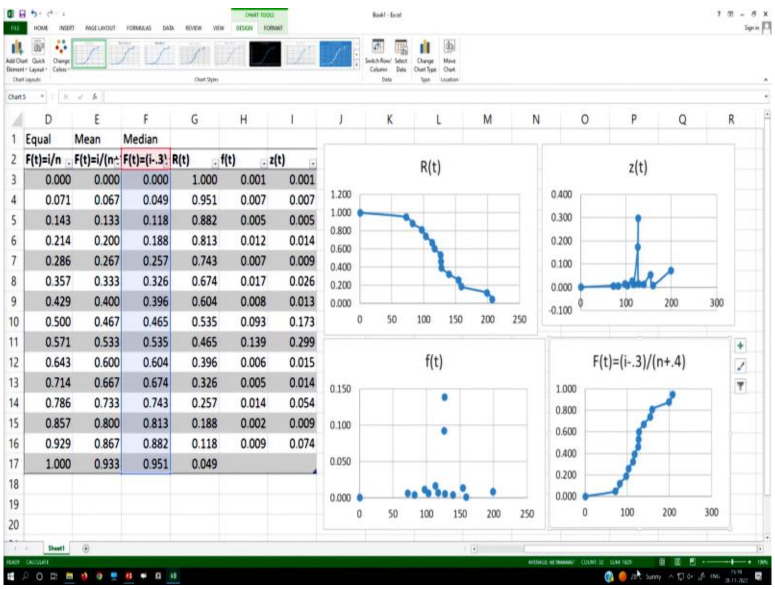
R(t)





So that will be kind of repetition also and that will also give you an idea that how we are doing the analysis in the so we will take only the data basic data. Let us say if we have the data. I am opening a Excel sheet here and making it little zoom, so that we can follow it up. I will use this only as the data point, I will not use any formatting here. So this is my i, i is the failure number. And this is my ti, ti is the time to failure which I have observed if I want, like, because it is 127.

One case was this, better, I will take a 126.75 and 127.25 that way I will have the balance average will be 127 for both. So here, we have this data. Now, we can use the different ranking methods here. So if I let us say, equal rank, if I use equal rank method, then we know ft is equal to i divided by n, n here is 14 here, so if I want, I can write somewhere here n is 14. Or I generally prefer to use a method, which is format as table, so I will use this here, now, ft, equal rank method that is equal to i divided by n, n is 14.

So I am putting that directly. If you see, I have got this ft value, I generally prefer to have all these values to have the same number of decimal points, I will use this function. 3 decimal points are enough for us to visualize this. Now, if I follow the mean rank method, for mean rank method, we know ft, fti is equal to i divided by n plus 1.

So what I will do that is equal to, what is my i, i is here divided by n plus 1 is 15. So, I will divide by 15. And again, I will do the format, same format I will use here. Then I will go with median ranking, for median ranking, ft is, is equal to, as we already know the formula, but I am still writing again, i minus 0.3 divided by n plus 0.4. Now this value, that means this is equal to i minus 0.3 divided by 14.4, because n is 14, so 14 plus 0.4 will be 14.4, the first value is always 0, it cannot never be less than 0. So we will put 0 here, rest of the values will remain same.

I will use again the formula format painter here. So now as we discussed, I want to use these values for reliability calculation. Now Rt will be nothing but 1 minus ft. So ft values already given me here, so I will use the same, I have got Rt here. Rt value, as per the median ranking. If I want to know Rt value as per the let us say, mean ranking, then I will use this formula.

And there will be little change here. If I want to know, use the equal ranking, I will use this as this is equal to 1 minus equal ranking unreliability value that is this. But as I discussed, we generally prefer to use the median ranking formula that is considered to be statistically more accurate. So we will use the median ranking formula. So this gives us the reliability value, we have got the reliability.

Now we want to calculate the failure density. So density function if you want to calculate ft, ft is minus DRT over dt or DFT over dt. So we know that this is equal to our DFT so that is next ft I am taking here minus previous ft, is this value and this divided by time difference, time differences this time minus previous time. So this is what I am able to get generally this will not have the last value, because that will be by default it is 0. So, it is counting the same value then we can get zt here zt as we discuss is ft upon Rt.

So, this is equal to ft divided by Rt this gives me this all the values. Now, let us see if I want to know how this reliability is changing or this ft is changing or zt is changing I can do the plotting here. So, let me show you if I plot the time versus let us say first I plot the ft value, reliability value I can go and do this plotting by going to the insert and choosing the XY scatterplot when I choose XY scatterplot I can choose the scatter or I can choose the scatter with line connections. So, I can choose this if you see this becomes my Rt, this has further functions, I can add more functions here, I can go to that say this is equal to ft, generally, I will not use here, ft here, I will use go with 1 here.

Let us say this becomes my Rt, so as you can see, I am able to see that how reliability is changing. So initially reliability decrement is not so fast but if you see after around 100 around us 70 hours or so, then reliability keeps on increasing and it is fastly decreasing then slowly little bit here. So, I am able to know I want to know the reliability at 100 hours I can calculate that this is almost here that is around 0.8. So 100 hours is somewhere here.

So that is somewhere between 0.81 to 0.74. So that comes out to be somewhere around 0.8 maybe. So looking at more closer data we will be able to get. Same way if I want I can plot ti versus let us say I plot the Ft, I will remove this data point, I will do it again. Because one data point is less. So here, generally, when I am plotting ft I would prefer to plot it as a step function where this value is given for the whole interval here. So, again I am just plotting for the showing here again, if you see my ft value looks like this, how so it shows that around somewhere here my number of failures are quite high and accept that number of failures are comparatively low.

So I have more failures concentrated in this region, similarly if I want I can plot the lambda t. But as I discussed, I would have preferably considered that my ti is plotted with fti as this constant value as a step functions. But step function plotting is not easy in Excel, I have to do certain modifications there, prepare format then only it will be done. So I will better show it as a then I will go to the chart type. And I will show this as the data rather than showing this change chart type I will go I will select this data type.

So you can see the data point of view, rather than lines, so here similarly I can plot the ti versus zt also. Same way I will go and this becomes my zt function. So as you see I am able to plot and zt if you see, I will go with the data type I will change chart type. Let us see how does it looks in line, line plot.

Again, line plot if you see it is kind of similar here. So we are able to now with the nonparametric data analysis if we have the complete data how to calculate the various probabilities reliability unreliability, unreliability also I can plot unreliability is given here, so I can just plot it. So, ft versus time versus ft, I will go with insert and I will again go with XY scatter, I will go with this remove this ti versus fti, okay some problem happened, so I will do this again.

I think some problem happened this is ti and this is ft. So, you see that this is my ft plot. So, here I can see various ways, so, if you see that ft is rising here then finally all failed, similarly, this I am saving and this sheet we will share with you maybe you know, whenever we get the chance, so fine. So, this is our complete data and that is the ungrouped.

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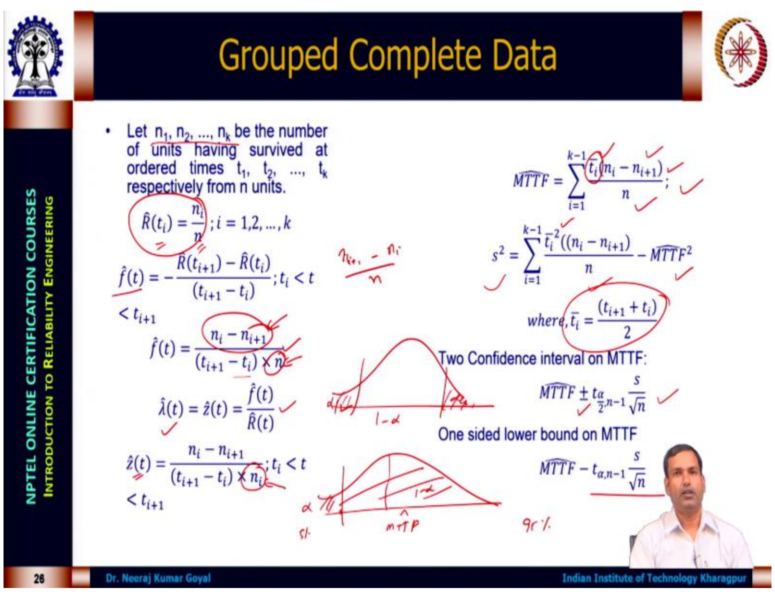

Now, let us go back to our presentation and see that what is next there, whatever we have done here same thing we can show it, see there. Now, there another type of data can be the complete grouped data. Grouped data means, you have intervals and like you have k number of intervals here and for each interval, you know the number of failures and so, n1 n2 nk means the number of units which have survived that means, at the start of zero interval how many units were there at the end of first interval, how much you data survive that means, n1 minus n2 will give you number of failures which have happened in second interval like that and t1 to tk is the time.

$$\hat{R}(t_i) = \frac{n_i}{n}; i = 1,2,\dots,k$$

$$\hat{f}(t) = -\frac{\hat{R}(t_{i+1}) - \hat{R}(t_i)}{(t_{i+1} - t_i)}; t_i < t < t_{i+1}$$

$$\hat{f}(t) = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i) \times n}$$

$$\hat{\lambda}(t) = \hat{Z}(t) = \frac{\hat{f}(t)}{\hat{R}(t)}$$

$$\hat{Z}(t) = \frac{n_i - n_{i+1}}{(t_{i+1} - t_i) \times n_i}; t_i < t < t_{i+1}$$

$$MTTF = \sum_{i=1}^{k-1} \frac{\bar{t}_i (n_i - n_{i+1})}{n};$$

$$s^2 = \sum_{i=1}^{k-1} \frac{\bar{t}_i^{\,2} ((n_i - n_{i+1})}{n} - \text{MTTF}^2$$

$$\text{where, } \bar{t}_i = \frac{(t_{i+1} + t_i)}{2}$$

So, if I want to know how much is the reliability for ti, so, that means, at ti ni numbers are surviving out of total number of units which have been put on the test. So, ni upon n gives you the Rt. So, ft will be 1 minus of this and small ft we will be change in Rt divided by Ti minus 1 so, that Ti is ni upon n this will be ni plus 1 minus ni divided by n. So, this will become same formula as we derived earlier divided by ti plus 1 minus ti divided by, earlier the number of failures was 1, because we were counting 111.

So, the change only here is that rather than one this has become the number of failures in the interval. At each time we are saying the how many failures are having happened, rest of the things are same, rather than 1 it has become ni upon minus ni minus 1 ni plus 1 and ni is the number of survival unit at time ti. So, number of failures would be difference in that and similarly zt lambda t, we are able to get and zt as we see whenever we are calculating ft out of total number of units on the put on the test.

And whenever I am calculate zt, the formula is same as ft, the change is that n is replaced with the ni because out of surviving unit how many units are failing in the interval that is only considered for zt, at the start of the interval. MTTF can be calculated in similar way for MTTF calculation we are taking the number of failures in the interval and the average time that means ti plus ti plus 1 divided by 2. So that is the middle point of the time. So that is Ti bar and then we have the M that is gives the MTTF. Same formula in a similar way.

So ti into number of failures will actually give you the total time for which the failure happened. So, cumulative time due to the failure. So, this is time total time in failure divided by n. Similarly, s square is also similar formula that ti square into ni minus ni plus 1 divided by n minus MTTF square and ti bar is how much that is ti plus 1 plus ti divided by 2 and we can also get the confidence interval MTTF this we can get from previous one also that is t alpha by 2 n minus 1 as by root n. So, that will be plus minus when we take this is the two sided confidence interval.

So, we have alpha by 2 this side and alpha by 2 probabilities lapped on this side and this is 1 minus alpha. And when we take one sided generally we are interested in lower bound. So, how much minimum reliability we are going to have or whatever is the minimum time to failure we will have so that MTTF.

So, this is the minimum that means this is 1 minus alpha. So, here alpha is left and this is nothing but the if this is our MTTF estimated value, this will be MTTF minus t alpha n minus 1 into s by root n. So, this is now if I take a 95 percent confidence level, in that case, this is will be 5 percent loss 5 percent will be left out. So this is the fifth percentile point for this data, you can use this. So, we will see that how we can use this data.

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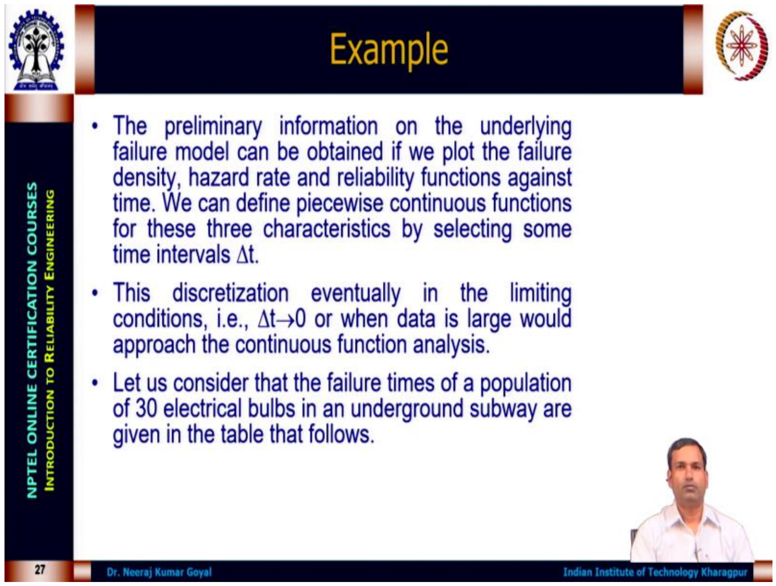

And how do we do the analysis for population. So, we will do the same as one more example in next discussion. Thank you.