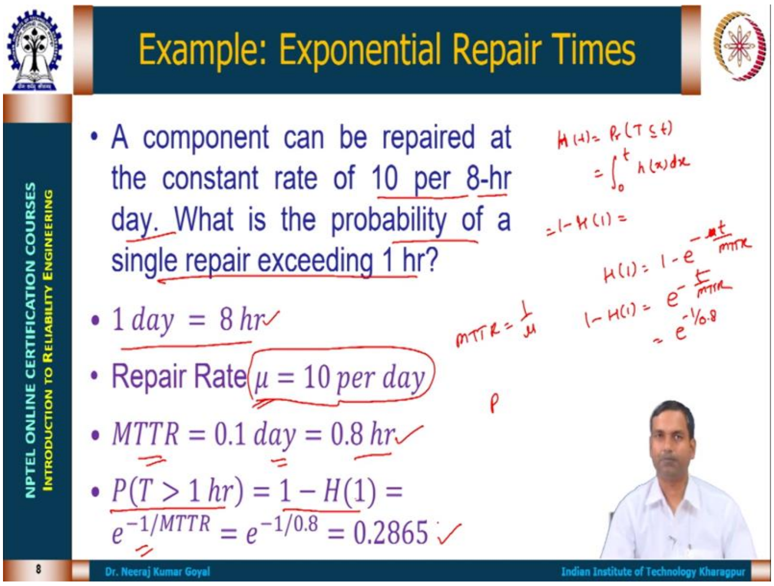
Introduction to Reliability Engineering
Professor Neeraj Kumar Goyal
Subir Chowdhury School of Quality and Reliability
Indian Institute of Technology Kharagpur
Lecture 37
Maintainability and Availability (Continued)

Hello everyone. In previous lecture we discussed about Maintainability. So, we will continue our discussion in the same directions.

(Refer Slide Time: 00:37)



$1 \text{ day } = 8\text{hr}$

Repair Rate $\mu = 10$ per day

$MTTR = 0.1 \text{ day } = 0.8\text{hr}$

$P(T > 1hr) = 1 - H(1) = e^{-1/MTTR} = e^{-1/0.8} = 0.2865$

So, we discussed that how maintainability can be calculated. Maintainability was given as maintainability is the H t we were writing here. So, H t is equal to probability that T is less than equal to t or we can say it is integration from 0 to t of h x d x. Now let us see how can we calculate the same for exponential distribution. So, if repair time is following the exponential distribution, can we calculate?

So, this is in a way will is going back to what we have done in the week 1 or week 2. Same thing what we are trying to do again. There we discussed the time to failure and reliability, here, we are discussing the maintainability. So, since, we have done in between data analysis etcetera and we are able to understand that how to get this mu etcetera. So, once we have this, so this mu we will estimate from the data. So, here it is expected that we have fit the exponential distribution to the time to repair data and we have found that mu is 10 per day.

And here the day is 8 hour's day that means in a dayit is 8 hours working only. So, if I say mu is mu means 10 that means 10 rapairs per day. So, 10 rapairs in 8 hours. What is the probability of single repair exceeding one hour. So, though repair rate is given 10 per day and what is one day one day is equal to 8 hours.

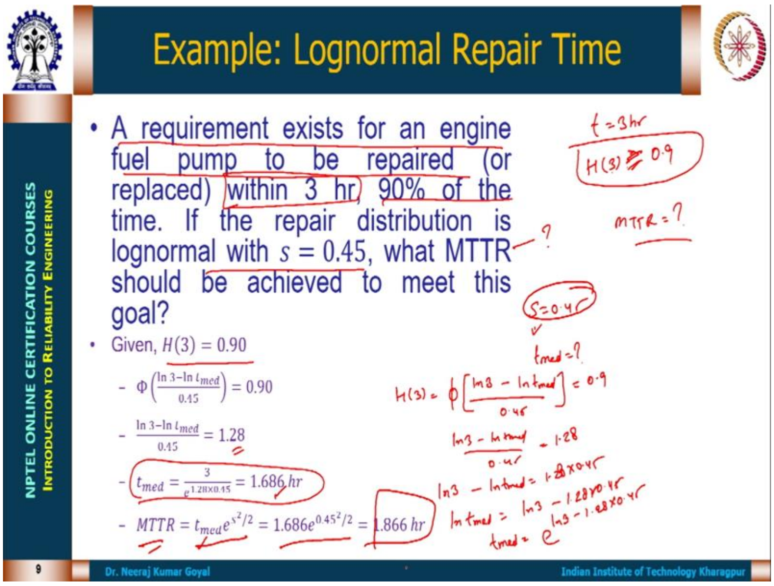
So, what is MTTR? MTTR is the 1 upon repair rate, for repaired or like reversing MTTF is equal to 1 upon lambda, same way MTTR is 1 upon mu. Mu is the repair rate here. So, that will become 0.1 per day. Now one day is equal to 8 hours. So, I can say 0.1 into 8 hours. So, 0.8 hours.

So, my MTTR is known to me, lambda is or we can say mu is known to me. So, I can know the probability. What is the probability? That single repair exceeding one hour that means it is 1 minus or maintainability. Maintainability is prepare probability of completing repair within one hour. So, my H t, H 1 is probability of completing repair in one hour. And what I am interested is?

I am interested in probability of not completing repair in one hour. That means my repair is exceeding one hour. So, that means I am interested in 1 minus H t probability that t is greater than one hour, 1 minus H1. So, 1 minus H1 means, we know that H 1 is equal to 1 minus e to the power minus mu t or we can say t divided by MTTR. Same thing we do it here.

So, I am interested in 1 minus H1. So, 1 minus H1 will be equal to e to the power minus t upon MTTR. How much t is here? Because H is 1 is here. So, it e to the power minus 1 divided by, how much is MTTR 0.8. So, that becomes, my value, once I solve this taking exponential of this, my value comes out to be 0.2865. So, that means there is a 28.65 percent chance that my repair will exceed one hour.

(Refer Slide Time: 04:18)



$$H(3) = 0.90$$

$$\Phi\left(\frac{\ln\ 3 - \ln\ t_{med}}{0.45}\right) = 0.90$$

$$\frac{\ln\ 3 - \ln\ t_{med}}{0.45} = 1.28$$

$$t_{med} = \frac{3}{e^{1.28\times 0.45}} = 1.686\text{hr}$$

$$MTTR = t_{med}e^{s^2/2} = 1.686e^{0.45^2/2} = 1.866\text{hr}$$

Though we have done one example for lognormal already, we will do it again here. That a requirement exists for an engine fuel pump to be repaired within 3 hour. So, our T is 3 hour, I am interested in 3 hours 90 percent of the time. That means my requirement is that H 3 is equal to is either greater than or equal to 0.9. So, if that means my H3 is 0.9. If repair distribution is lognormal with s equal to 0.45.

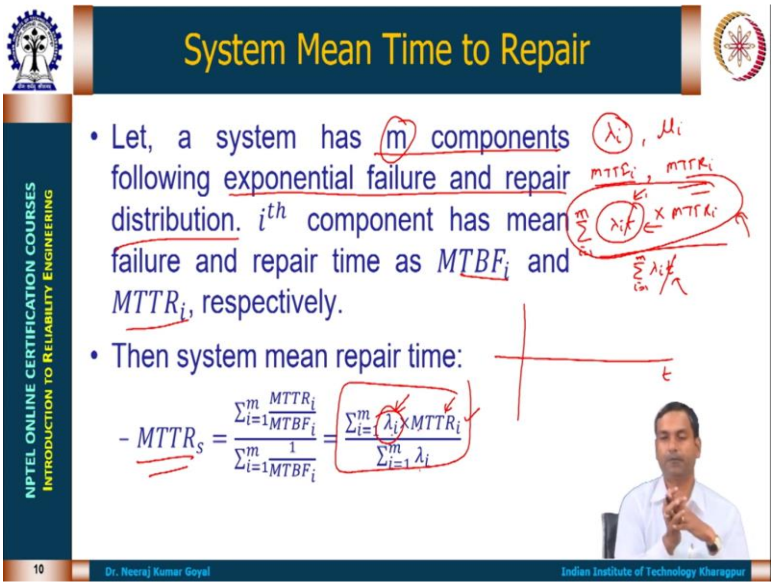
So, my if given is that my I am getting the 90 percent repair completed in 3 hours. So, that means my maintainability is for 3 hour is 0.9. So, my S value is given here 0.45, I want to know how much is the MTTR. So, we know that for lognormal distribution there are two parameters one is s another is t median. Here, t median is not given to us. So, first to know the distribution we have to calculate the t median. How we can calculate the t median?

We know what is H 3 is equal to standard lognormal cumulative distribution of Ln of 3 minus Ln of t median divided by s, s is 0.5. And H3 is how much? That is given to us is 0.9. So, now I can take the phi inverse of 0.9, phi inverse of 0.9 is 1.28. So, Ln of 3 minus Ln of t median divided by 0.45 is equal to 1.28. So, Ln of 3 minus Ln of t median would be equal to

1.28 into 0.45 or I can say Ln of t median would be equal to Ln of 3 minus 1.28 into 0.45 t median I can get it as exponential of Ln of 3 minus 1.28 into 0.45. This comes out to be this value, 1.686 hour.

Now, once we get this t median, I can get the MTTR. MTTR formula is known to us that is t median into e to the power s square by 2. I know the t median also and I know the s value also that is 0.45. Once I put the these in the formula I can get the MTTR that is 1.866 hours. So, whether generally, we will see that most of the time MTTR either is considered to be following the exponential distribution or the lognormal distribution. These are the two prominent distributions which are used for the time to repair, while for failure data we consider exponential or viable mostly. So, we can use this.

(Refer Slide Time: 07:57)



$$MTTR_S = \frac{\sum_{i=1}^{m} \frac{MTTR_i}{MTBF_i}}{\sum_{i=1}^{m} \frac{1}{MTBF_i}} = \frac{\sum_{i=1}^{m} \lambda_i \times MTTR_i}{\sum_{i=1}^{m} \lambda_i}$$

Now let us say, we have a system. In our system we have m components. Now, every component will have its own repair time, all components are not going to be repaired in the same time. So, let us assume that all the systems are having the exponential failure in repair time.

That means time to failure also follows exponential distribution and time to repair also follows the exponential distribution. So, we can say that i'th component is having failure rate as lambda i, repair rate as Mu i or we can say, mean time to failure as MTTF i and mean time to repair is MTTR i. This is MTTF i MTTR i. Now, I want to know the system mean time to repair. Now, what happens here?

Now, the system when we want to find out the repair time. So, system repair time would be the time spent in repair. But what happens? Let us say for a system, system can fail due to any of this component failure. But the system will fail more according to those components which are failing faster. That means the component which are having higher lambda, they will feel more.

So, the repair time taken in that component which is having high failure rate will have the higher contribution in the system repair time and the components which are failing lesser, their repair time would be having the lesser contribution in the repair time of the system. Because system is not failure in those failure modes.

So, whichever failure happens based on that repair time will be taken in the system repair. So, system repair time is actually weighted sum of the repair time and the weight is decided by the failure rate. So, a system which is having higher failure rate, their repair time would be having the higher contribution to the system repair.

And the system which, the components which are having lesser repair time or lesser failure rate, they will be having, because in the total duration of the system life, the number of failures will depend on the failure rate. The system which having high failure rate, number of failures will be high.

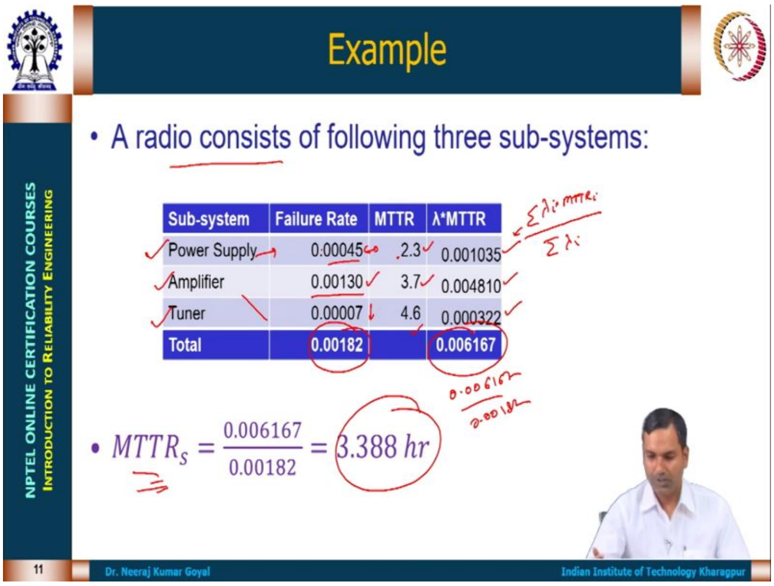
So, if we say that, if we have the time t then each component will fail lambda i t, that number of times it will fail is lambda i t, in the total time T. And every time it fails, you have to spend the time MTTR i. So, per unit time, if you want to calculate this. So, and in time T, how many failures you are total having?

Total failures you are having is the summation of lambda i t. So, i equal to 1 to m, m here and i equal to 1 to m here. So, this is the, if you see here, you know this time is the total time spent in repair because this gives the number of failures due to each component. So, for component i repair time spent is lambda i t into MTTR i and for all components it will become summation of this time.

So, in total time T, this is the time which you are spending in repair. But we want mean time to repair, that means per fear how much time it is taking. So, how many failures are happening? Failures are happening is for one component is lambda i t. So, for n component is summation of lambda i t. Now, this t and t get canceled. So, this will become lambda into MTTR i divided by summation of lambda i.

So, this becomes our system repair time. So, as we see here, the system repair time you can control by assigning lower MTTR to the high lambda. And you may allow High MTTR for lower lambda. So, if lambda is low, high MTTR will also have the less contribution. But if lambda is high then you have to keep MTTR to be smaller. By keeping MTTR to be smaller, you are ensuring the system MTTR is smaller.

(Refer Slide Time: 12:35)



$$MTTR_s = \frac{0.006167}{0.00182} = 3.388\text{hr}$$

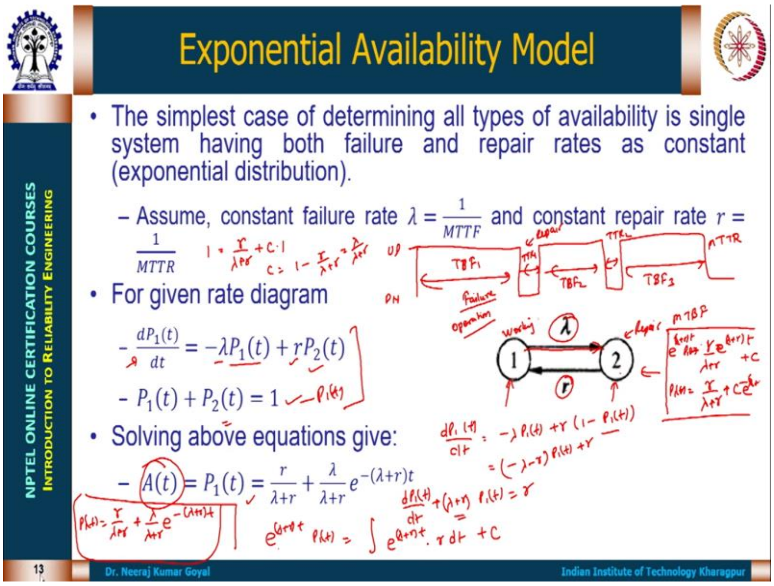
Let us take this as an example. So, if we consider one radio which is having power supply, amplifier, tuner, three parts are there. Now, power supply has the failure rate 0.00045 and amplifier has the failure at 0.00130 and tuner has the 0.00007. As you see this is the highest filler rate and this is the lowest failure rate and this is somewhere in between.

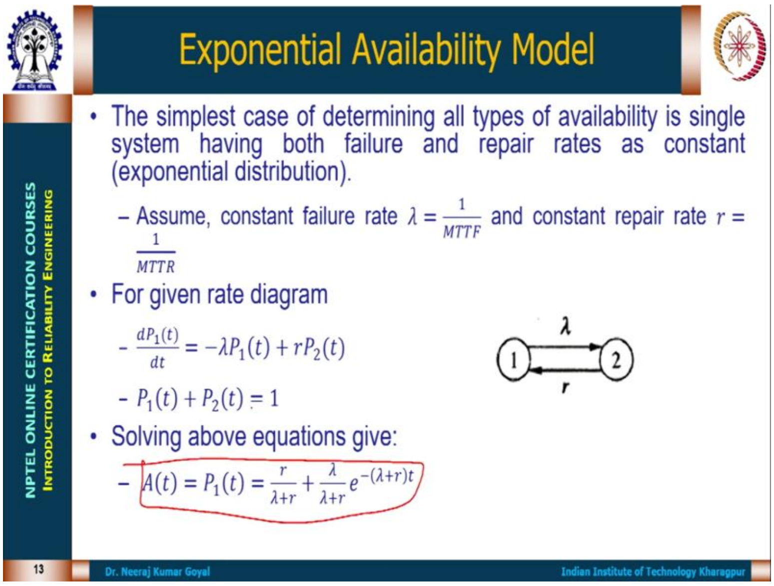
Now, if you see that the time to repair for power supplies 2.3 and all is in hours, amplifier is 3.7 and tuner is 4.6. So, how can we calculate the MTTF of system? MTTF of system we can calculate by multiplying failure rate with MTTR. That is power supply that is amplifier, that is tuner.

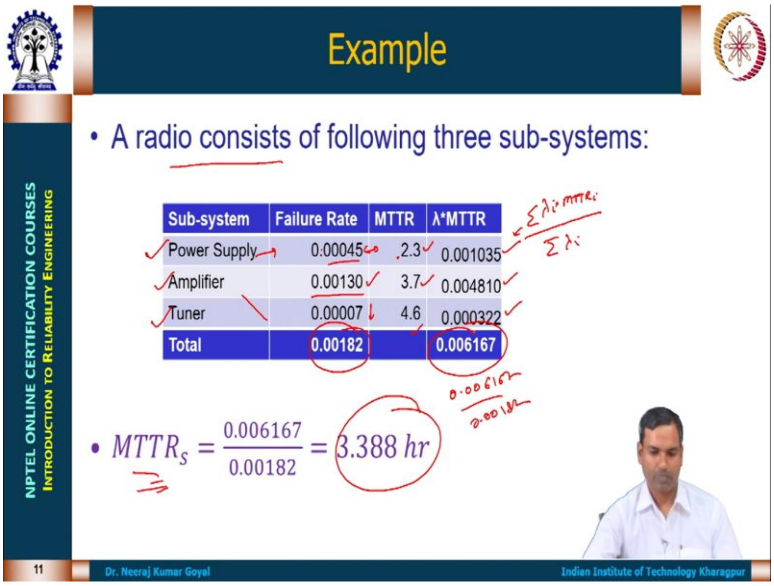
So, this becomes lambda i MTTR i. If you take summation of this and divide by the summation of lambda i, we get the MTTR of system. And summation of lambda i is this, summation of this is 0.006167 divided by 0.00182 and that gives me the 3.388 hours. So, this is a very simple way. Many times we are not sure and we want to know the system MTTR.

So, at the system level, this is a very simple formula we can use for calculating the mean time to repair. If you know the mean time to repair for each and individual part of it. But along with mean time to repair, we should also know the failure rate of the each part then only we can calculate the system MTTR. Because repair only comes into the picture when failure happens. So, it is dependent on the failure time.

(Refer Slide Time: 14:37)







So, next as we discussed earlier, we discussed the maintainability, we discussed the availability. So, we if we start discussing about the availability now then as we discussed earlier availability is considered to be like this. Where or I will do this diagram again because generally what will happen?

A system is up initially. It will work for certain time then it will fail that will be repair then it will be again up then again prepared both are random variable this distance may not be always same this distance will also not be same. So, this is up, this is down, and this is our repair and this is our failure or operation.

So, here we have the time to failure 1 we can say this is time to failure 2, this is time to failure 3 or we can see time between failure also. Generally, we use time between failures, so

I will use TBF here. And this is TTR1, time to repair 1; this is TTR 2, like that. So, if you take average of TBF, we will get the MTBF, if you take average of repair, time to repair, we get MTTR. All these concept we have already discussed and we have already seen how to calculate them.

Now let us discuss that if we want to calculate availability then I will first revise you the different types of availability which we discussed earlier, then this exponential availability model we derive from here. We see here that system can be in two state, system or component whatever we are considering can be in two states.

This is the working state or operation and this is the repair, failure or repair. When it fails, it goes to the repair. So, what happens when system is working, the failure rate is lambda. So, it can by the rate of departure from working state is lambda. Once it reaches to the state two in state 2, it is in failed condition, it is under repair and the repair it is r.

So, the rate by which it can come back again to working condition is r. So, this makes our Markov diagram. Now, this Markov diagram we can solve. How can we solve? As we discussed earlier, for Markov diagram dp1 over dt, dp1 over dt is a negative of outgoing and positive of incoming, outgoing is lambda, lambda from state P1 minus lambda P1 t. Our incoming is positive that is r into P2, r into P 2 t.

Similarly, here generally, as we discussed earlier, whenever we are solving these equations, we do not want to use all the equations. Like for one state equation we will replace with our absolute equation which says that total probability for all states is one. So, rather than developing for p 2 because if I am having that thus P2 state becomes obviously coming from the first state.

So, one equation which we also always need to use is p 1 t plus P 2 t is equal to 1. So, we have the two equations here. Now, if we solve these two equations, we will get this. How can we solve this? Here, we have the P 2. So, this P 2 we can resolve, like d p 1 t over d t is equal to minus lambda P 1 t. This also we can turn it into the P 1 that is plus r into 1 minus P 1 t.

So, this will become minus lambda plus r. I will erase this. So, minus lambda P 1 t and from here again minus r will come. Minus r into P 1 t plus r. So, if we take it on left hand side, this will become d p 1 t over dt plus lambda plus r into P1 t is equal to r. This equation we have solved earlier also, similar equations. How do we solved? We took this as the multiplying factor.

So, this becomes e to the power lambda plus r into t P1 t is equal to integration of e to the power lambda plus r into t into r d t plus C. When we integrate this, r is constant, so, this will become, I am taking it here. So, this will become right hand side I am taking it here. So, right hand side is 1 upon lambda plus r e to the power lambda plus r into t into r. So, r I am taking here plus C.

And left hand side is e to the power minus e to the power lambda plus r into t into P 1 t. So, P 1 t if I calculate that will be equal to, this will go here. So, this will become r upon lambda plus r plus C into e to the power minus lambda plus r into t. Now, this as we know at time t equal to 0, system is in state 1.

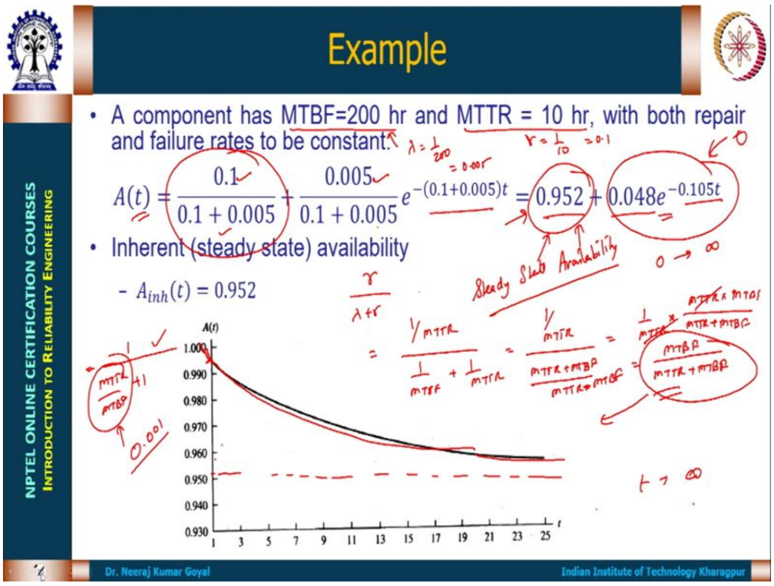
So, if I put t equal to 0, P 1 t will be equal to 1. 1 is equal to r upon lambda plus r plus C into t equal to 0 e to the power will become 0. That will be 1. So, c will be equal to C will be equal to 1 minus r upon lambda plus r. So, same thing we can write. So, here my equation will become, I am the removing some space here. I will use this space here.

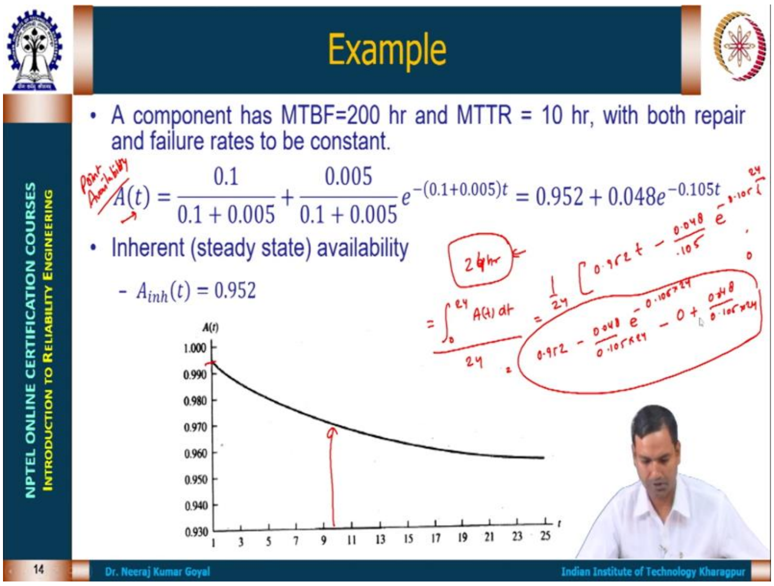
So, here the same equation P 1 t. P 1 t is equal to r upon lambda plus r, plus 1 minus r upon lambda plus r will be lambda plus r minus r. So, that will become lambda upon lambda plus r, so, lambda upon lambda plus r. C is equal to and multiply by e to the power minus lambda plus r into t.

So, if you see this is the same equation P 1 t. P 1 t is r upon lambda plus r plus lambda upon lambda plus r e to the power minus lambda plus r into t. So, like few steps when we follow, we are able to derive and we are able to get the P 1 t. Now what is availability here? Availability here is that system is in working state.

So, the probability that system is in working. So, that means the probability the system is in state 1. So, P 1 t is nothing but the availability here. And what is the unavailability? Unavailability is the probability that system is in state 2. And what is P 2? P 2 is 1 minus P 1 t. So, if I subtract this same value from 1, I will get the unavailability, that is probability that system is in state 2. So, as we have seen here, we get this A t equation that is the that is my this equation.

(Refer Slide Time: 23:11)





If I let us say if I plot it here. I have plotted it here. Let us say for an example, if I take MTBF is 200 hour and MTTR is 10 hour. So, that means lambda is equal to 1 upon 200 and r is equal to 1 by 10. So, that is equal to 0.005 and this is equal to 0.1. So, I will apply this formula r upon lambda plus r that is 0.1 divided by 0.1 plus 0.005 plus 0.005 divided by 0.1 plus 0.005 into exponential of 0.1 plus 0.005. So, this will become 0.952 plus 0.048 e to the power minus 0.105 into t.

Now, if you see here how this equation will look like. Here we have the constant value 0.952. So, somewhere here 0.952, if you draw a line, this is the constant line like A t value will never be below this because this value, exponential value can be some value from 0 to infinity, it cannot be negative.

So, this value cannot be negative. Since this cannot be negative, my this value is always going to be 0.952 or higher than 0.952. That means my minimum availability is 95.2. And how this will vary? 0.048 if you add initially t equal to zero, 0.952 plus 0.0, it will be 1. Initially, it will be starting from 1, at t equal to zero.

Then it will be exponentially decaying and that is defined by the e to the power minus 0.0 t, 0.105 t. And this decay will be continuing and this will become asymptotically touching to the 0.950. So, as we said that time t is very large or t turning to infinity, what will happen? My value for the availability will be this. So, this availability we call it as steady state availability or inherent availability. And what is the formula for this steady state availability?

That is this. What is this? This is r upon lambda plus r. And what is r? r is 1 upon MTTR. And what is lambda? 1 upon MTTF. And what is r? 1 upon MTTR. That is equal to1 upon MTTR, this divided by MTTR plus MTTF or M T B F, we can say. M T B F divided by MTTR into MTBF.

This if we solve, this will be equal to 1 upon MTTR into MTTR into MTBF divided by MTTR plus M T B F. This will be equal to MTBF divided by MTTR plus MTBF. As you see this is the popular formula. Whenever you say availability, what is availability? Availability is MTBF divided by MTTR plus MTBF. This is how it comes. This is the steady state availability. Because what happens?

When t is infinity, this value will be zero. And only value remaining will be 0.952. So, steady state availability is actually coming from the how much time we are taking for repair and so, if our repair time is small, if MTTR is small, what will happen? This value will tend to become 1.

So, if we can repair fast then we can make the system available for uses for a longer period because even though failure happens, we are able to repair fast. So, iti s depending on the how much time we are taking for failure and corresponding to that how much time is taken in the repair.

So, ratio of repair to failure will be deciding. If in other way, let us try, if I divide by MTBF then this will become 1 upon MTTR divided by MTBF plus 1. So, if you can work on this ratio, if you make it almost near to zero, availability will be 1. If you make this ratio, let us say as low as possible, if you make it 0.001 it will be almost similar to very near to 1.

That means by reducing the maintenance time or repair time, you are able to have achieve the higher availability. That is the steady state availability. That means that is the minimum availability you will be getting from the system after the when the repair is, when you are expecting that failure will happen.

But the case here is that we are discussing the TTR and TBF are following the exponential distribution. But this is what a commonly used formula is and this is widely used for the purpose. And the availability A t which we have calculated here, this a t this A t is called point availability. Why it is called point availability?

Because it is giving you at any moment of the time, I want to know the availability I can know. So, this is function of time, it is depending on the time. So, it is considering the newness into the picture and when system is new, availability will be higher, when system is old; availability will be lower even though we are doing the repair.

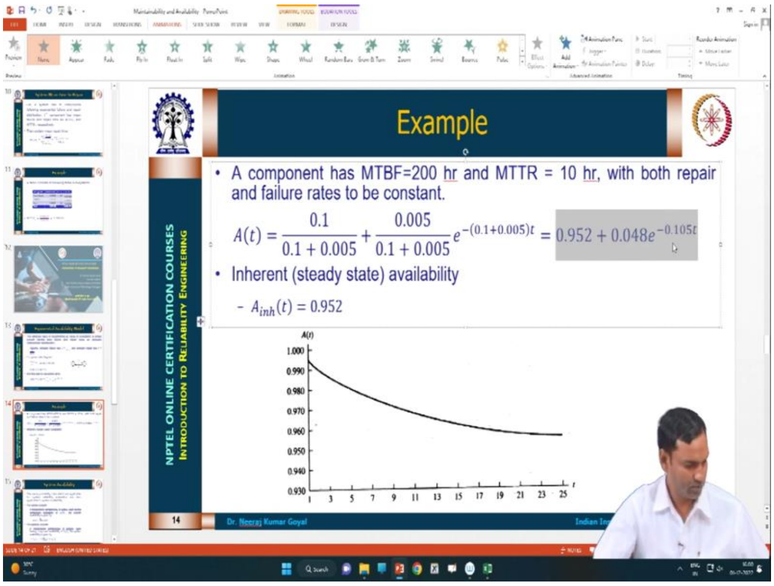
Similarly, if we want to know that what is my availability here for let us say interval availability. I want to know that for 20 hours of operation that my objective is I want to use the system for 20 hours for 20 hours of operation or let us say 24 hours of operation how much is my average availability or interval availability. So, what I can do?

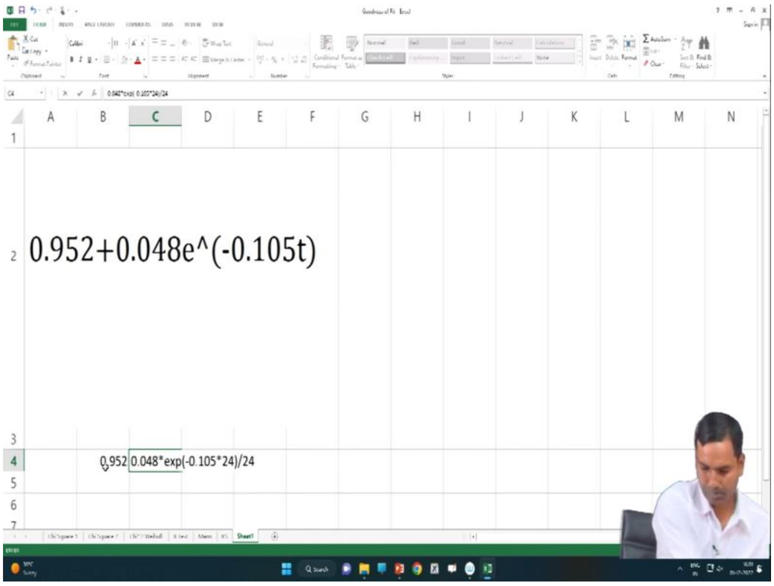
I can integrate 0 to 24 A t d t and divided by the 24. So, that will be 1 by 24 integration of A t d t if I do then 1 by 24 or A t is 0.952. If I integrate this with d t it will become t minus 0.048 divided by 0.105 e to the power minus 0.105 t. So, I can calculate this 0 to 24. So, when I put 24, this will be equal to 24 into 0.952. So, this will become 0.952 then again when I put or minus 0.048 divided by 0.105 into e to the power minus 0.105 into 24.

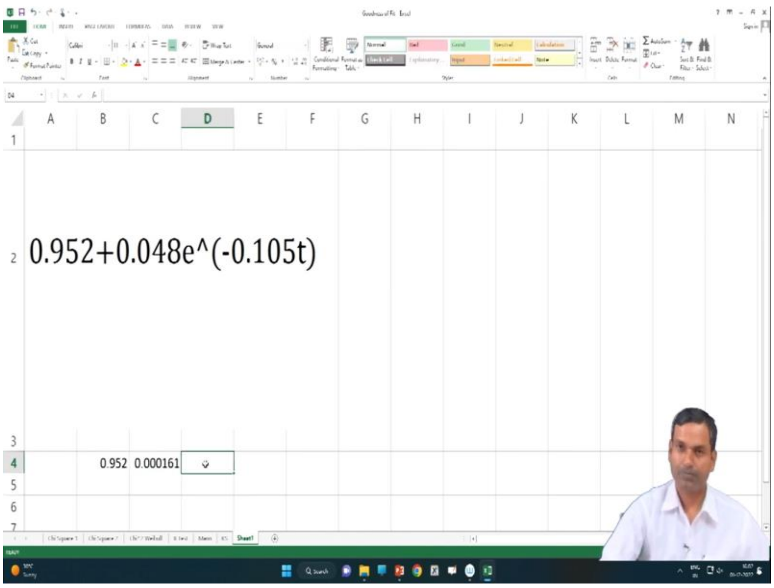
Then I will put the t equal to 0. So, minus when I put t equal to 0 this will be 0 and this will become plus. When I put t equal to 0 then this is e to the power minus 0 will become 1. So, this will be 0.048 divided by 0.105 and this 24 will also be coming here. Here also it will come, here also it will come. So, this if we solve, we will be able to know the interval availability.

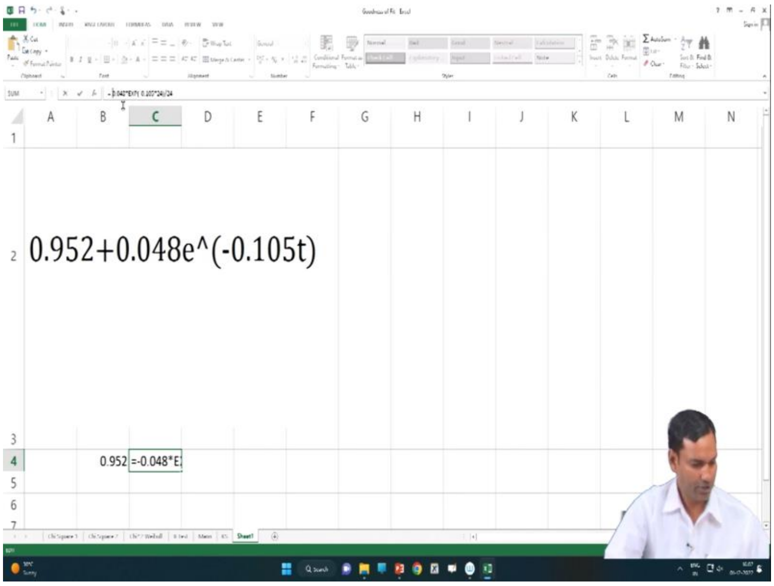
That means what is the probability that I will find the system or what is the proportion of the time my system is going to be in working condition in 24 hours of operation. If you wish, we can let us try to calculate this. I am discarding this. Let me because this is obvious what we got here. So, may be can use Excel here.

(Refer Slide Time: 32:17)

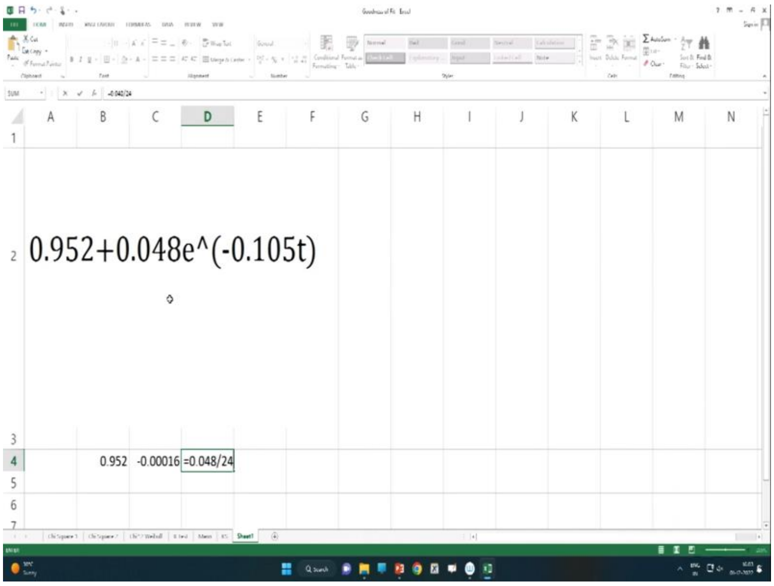




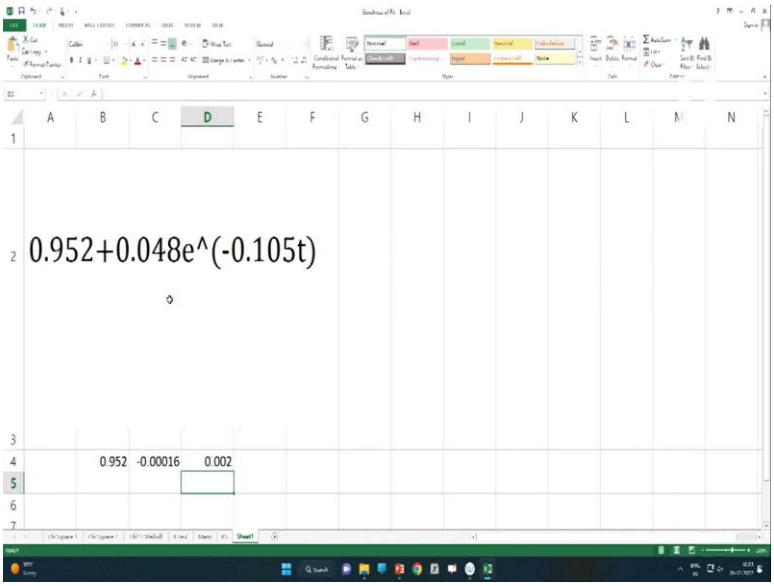




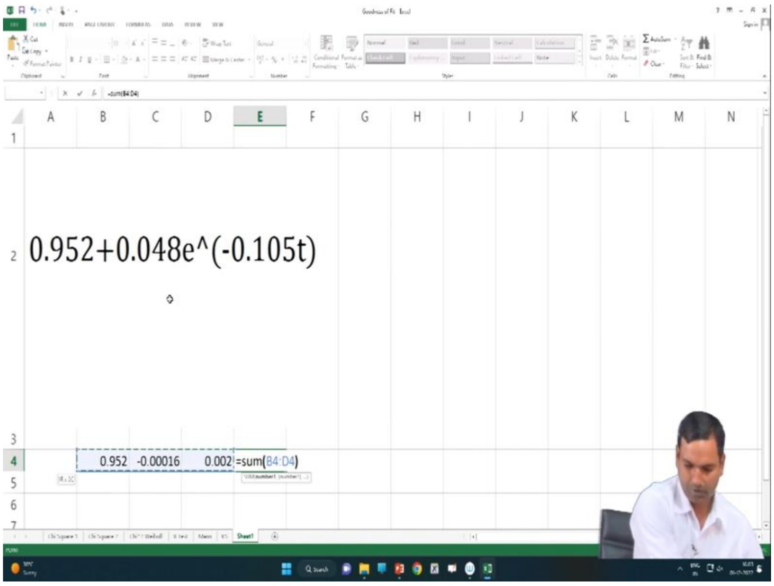
0.952+0.048e^(-0.105t)
0.952
=-0.048*E

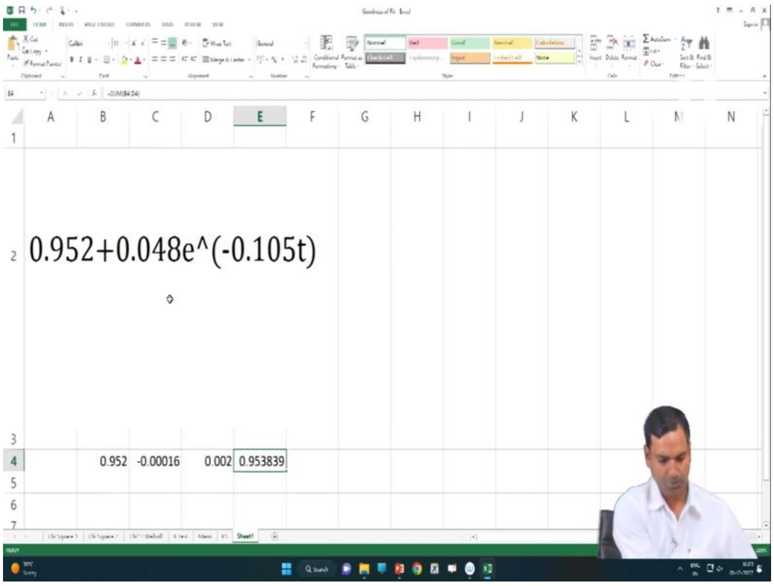


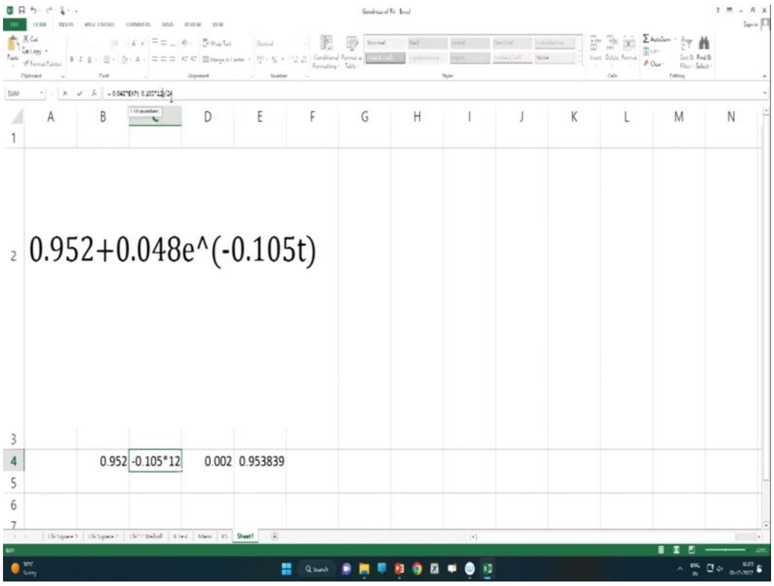
0.952+0.048e^(-0.105t)
0.952
-0.00016
=0.048/24

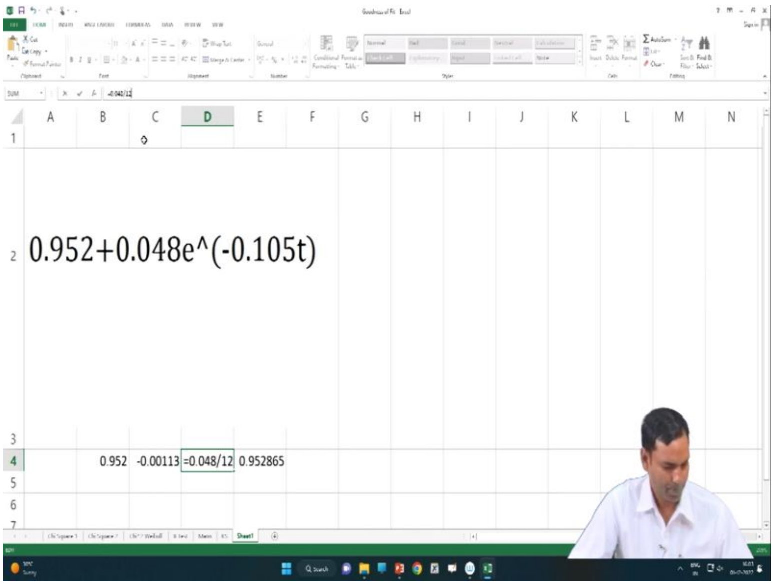


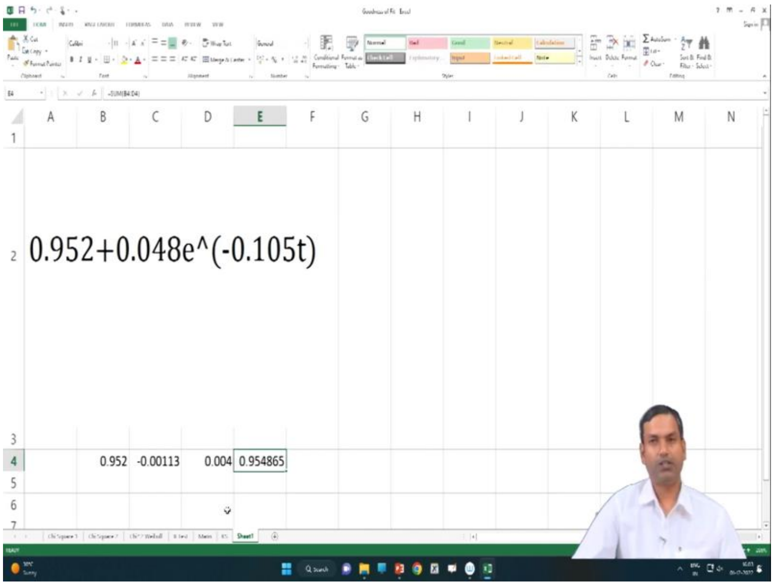
0.952+0.048e^(-0.105t)
0.952
-0.00016
0.002

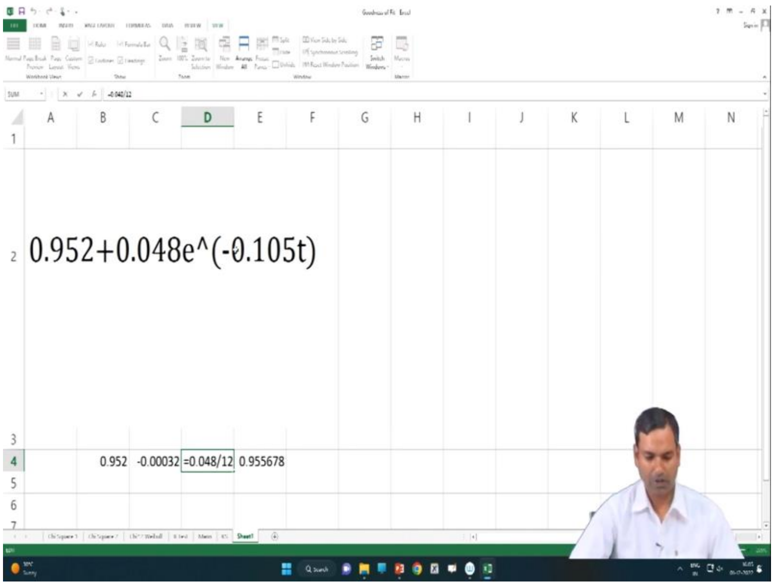

0.952+0.048e^(-0.105t)
0.952
-0.00016
0.002
=sum(B4:D4)

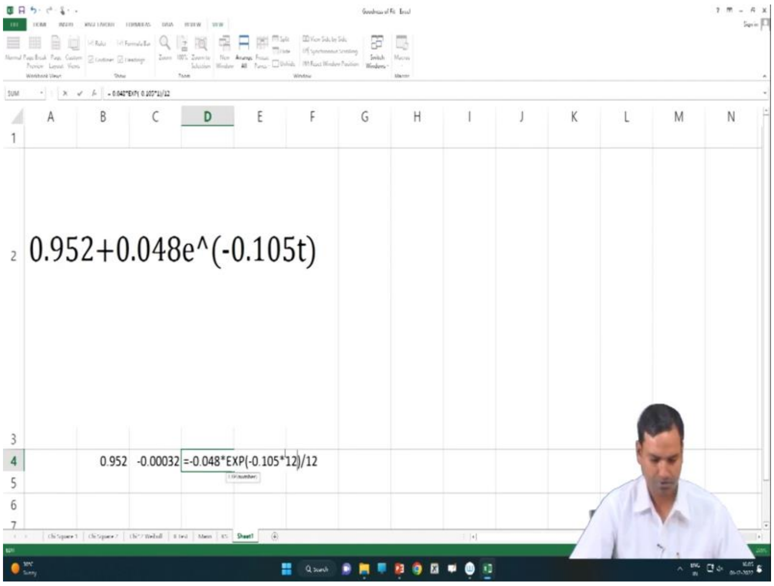

0.952+0.048e^(-0.105t)
0.952
-0.00016
0.002
0.953839

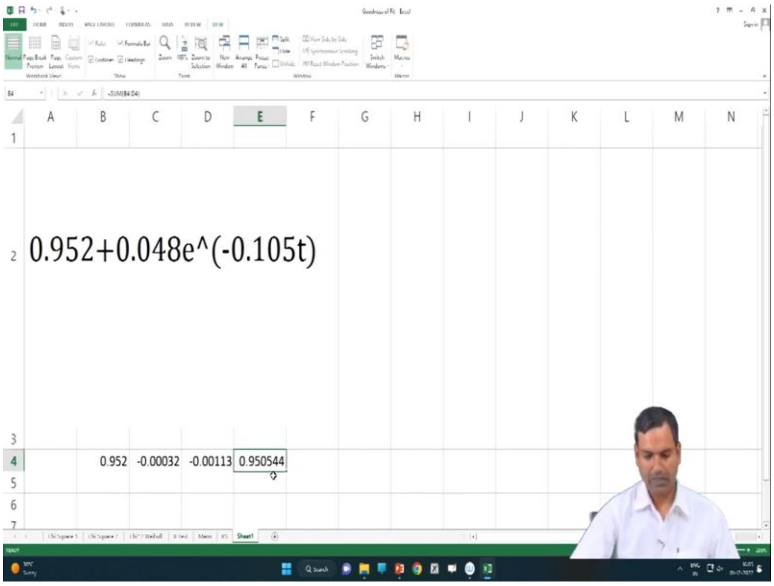

0.952+0.048e^(-0.105t)
0.952
-0.105*12
0.002
0.953839


0.952+0.048e^(-0.105t)
0.952
-0.00113
=0.048/12
0.952865


0.952+0.048e^(-0.105t)
0.952
-0.00113
0.004
0.954865


0.952+0.048e^(-0.105t)
0.952
-0.00032
=0.048/12
0.955678





I will use the same file just for showing. My availability or my lambda was, so, as I calculated here that was 1 by 24 into 0.952. So, 0.952. I will just copy this here so that, yeah. So, when I integrated this. Then first term came as 0.952 as it is. Then second term when I put 24 there then this became or 0.048 into exponential of minus 0.105 into 24.

And 24 hours we have to divide for all the data. So, this will be. So, as we saw that 0.952 was coming for first term. And for second term we were getting this, when we put t equal to 24. Then when we put t equal to 0 then first term was becoming 0 and second term minus minus became plus. And this was negative term.

So, I will put negative sign here. And positive term was coming to be 0.048 divided by 24 because 2 4 is the hours. So, how much is my availability here? Availability here is sum of these three terms and that turns out to be 0.953. So, that means I can say for 24 hours my availability is 95.38 which is definitely larger than my steady state availability 0.952 but it is less than one.

So, here whatever is the period, let us see, if I am interested in 12 hours period, then for 12 hours period, this 24 I will change it into 12 then for 12 hours period if I want to know the availability then this will be my availability. If you see, my availability has become more in because in initial 12 hours my system is supposed to work more because chances of failures are smaller.

If I want to know the availability for let us say12 to 24 hours, if I want to do that then same formula, what will happen? I am using pan here, pan is not coming. So, in that case as we discussed earlier, what will happen? First we will write 24. So, we will multiply by or 24 then subtract again because of the t, it will come again 12. So, this value constant value will come as it is.

This value will show little change like when we initially we will to take for 24; we are going to divide by 12 because division factor is 24 minus 12 that will remain 12. But the second term is also going to be divided by 12 only. But this what will happen here? Here, rather than 0, because e to the power is not 0 here, e to the power I have to take is 12 here because the second term I have to take is 12 and then divided by 12.

So, if you see that for12 to 24 hours, I want to know the availability between 12 to 24 hours of operation, that comes out to be 0.95, while I think little it was little larger when I consider the 0 to 12 hours of availability. So, same 12 hours when I has the initial 12 hours and second was the 12 hours I have taken after 12 hours. In both the cases availability is little different.

But if you see that if I taketime as let us say for 100 hours to 112 hours, then another time if I take the 110 to 124 hours 122 hours. So, that 12 hours difference there will not make much difference because the newness of the system is gone. So, because of that availability will appear to be similar.

So, when t is becoming larger. When time becomes larger, the change in availability will not be that much, and the availability would appear to be very similar to 0.952. Only for initial stages of system operation, it may show little change in the availability. So, we will stop our discussion today here. We will continue our discussions about availability in our next classes. Thank you.