Introduction to Reliability Engineering
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Lecture 06
Constant Failure Rate Model-I

Hello everyone, we have completed one week of lectures, and now we have moved on to the second week. Our focus this week will be on constant failure rate models, also known as the exponential distribution. In the previous week, we discussed various reliability indices, their definitions, and their basic relationships. This week, we will be focusing on the exponential distribution, including how it is used for modeling and how it can be used to evaluate reliability.
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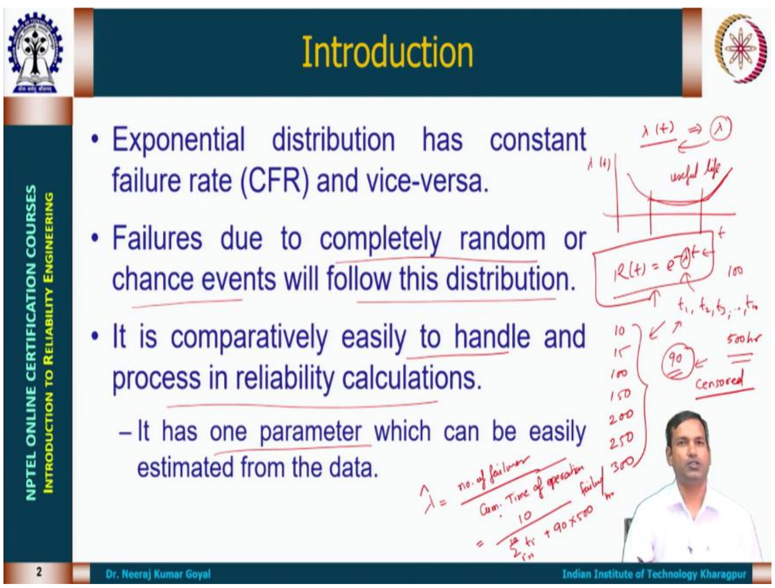


As discussed in previous lectures, the exponential distribution is characterized by a constant failure rate. This failure rate, denoted as λ(t), remains unchanged over time. Therefore, if the failure rate remains constant, the default model becomes the exponential distribution.

Failures can occur due to wear and tear, and the exponential distribution finds application in such scenarios. As we discussed earlier, the bathtub curve represents λ(t) and t, where λ represents the failure rate during the useful life of the component. In the case of the exponential distribution, λ is constant during the useful life of the component. This constant rate is significant because it represents the period during which the component is expected to function correctly without any manufacturing or design defects or degradation.

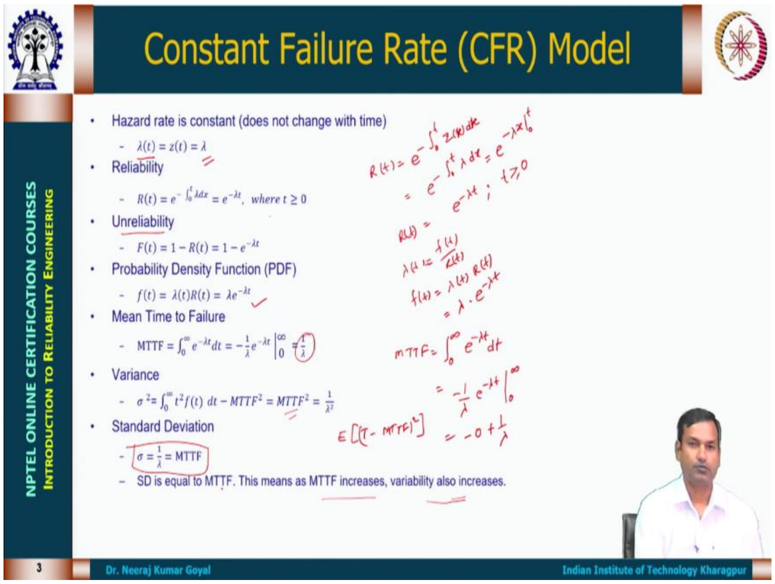
The reason why failures occur randomly or due to chance events is that any component can fail for these reasons. Therefore, there is no specific assignable cause to be found, and this can happen to any device. Hence, any device can fail, and the failure rate follows the exponential distribution. Since the exponential distribution is a one-parameter distribution with various properties, it is easy to handle and can be used in most relative calculations. When referring to system reliability standards, if no distribution is mentioned, it is assumed to be an exponential distribution.

The exponential distribution is so popular that reliability has become synonymous with it. The reliability R(T) is equal to e^(-λ(t)), which is only applicable for exponential distribution. However, other distributions are also used, but they are more constrained to the data analysis part. Specifically, for reliability prediction data, the assumed distribution is almost always the exponential distribution. When collecting and analyzing data, other distributions with a non-constant failure rate may be used, but due to its popularity, the exponential distribution is used almost everywhere. This distribution has only one parameter, which is time, t, and the function of lambda is the only parameter.

Lambda, which is the parameter, can be easily evaluated from the data for reliability. Reliability data is generally the time to failure, like t1, t2, t3, because the random variable is the time to failure. For example, if we put 100 devices on a test and had ten failures in 500 hours, the times may be recorded as first failure at 10 hours, second at 15 hours, third at 100 hours, fifth at 150 hours, sixth at 200 hours, and seventh at 250 hours, and so on. We may have various data points listed as t1 to tn. Out of the 100 devices, 90 devices have not failed, so they have worked for 500 hours. Lambda can be easily calculated as the number of failures divided by the cumulative time of operation. In our example, the number of failures is 10, and the cumulative time to failure is the summation of ti, where i ranges from 1 to 10, and for the remaining 90 devices, it is 90 multiplied by 500 hours.

This formula is only valid for the exponential distribution, but it is so easy to understand that the industries assume it whenever there are multiple failures, but many devices have not failed. These are called sensor devices because they have not failed, and we do not know when they will fail. However, we know they will fail after 500 hours since they have not failed yet. Using this information, we can easily calculate lambda, which can help us evaluate other parameters. The same is not the case when considering time-dependent failure rate models.

In this week, our focus will mostly be on discussing the constant failure rate models. After completing the constant failure rate models, we will discuss time-dependent failure rate models. (Refer Slide Time: 07:46)



In case of constant failure rate models if we see λ(t), but that is over hazarded it is constant. Now, we know from the formula that R(T) is equal to given us e to the power minus integration from 0 to t, z(t) dt or lambda t dt, this formula or lambda x dx this formula we have already discussed in previous classes. So, as we see this comes out to be e to the power minus, now, this is constant. So, let us see constant values lambda and dx. So, this will be equal to e to the power minus x from 0 to t and lambda this will give us e to the power minus lambda t because, when we put 0 that will become x t minus 0 so, that will become t.

$$\lambda(t) = z(t) = \lambda$$

$$R(t) = e^{-\int_0^t \lambda dx} = e^{-\lambda t}$$

So, this is our R(T) formula what is, so, this is how we get the R(T) as we know this distribution is all applicable only for t greater than equal to 0 time is not considered to be negative quantity here. Similarly, we can calculate unreliability, unreliability means we can also say it is the failure probability. Failure probability is nothing but 1 minus reliability. So, that is 1 minus e to the power minus lambda t. Probability density function for the same we can evaluate as we know earlier formula that lambda t is equal to f(t) upon R(T).

$$F(t) = 1 - R(t) = 1 - e^{-\lambda t}$$

$$f(t) = \lambda(t)R(t) = \lambda e^{-\lambda t}$$

So, we can get f(t) from here f(t) will be equal to λt into R(t). Now, lambda t is constant here that is lambda and R(t) we know that is we have already calculated $e^{-\lambda t}$. So, the same becomes PDF here. For calculation of mean time to failure, as we know mean time to failure MTTF is equal to integration over full range. Now, t is not negative t can be only from 0 to infinity RTDT, what is R(t)dt? R(t) is $e^{-\lambda t}$dt this if you integrate this will become minus one upon lambda, e to the power minus lambda t from 0 to infinity.

$$\text{MTTF} = \int_0^\infty e^{-\lambda t} dt = -\frac{1}{\lambda} e^{-\lambda t}\Big|_0^\infty = \frac{1}{\lambda}$$

Then we put limit equal to infinity this will become minus 0, minus minus will become plus t equal to 0 putting it will give the 1 upon lambda. So, as we see, MTTF comes out to be 1 upon lambda. So, or we can say that, MTTF is inverse of the parameter only parameter is the failure rate here constant failure rate that is lambda. Similarly, we can evaluate variance, variances nothing but the expectation of t minus MTTF whole square expectation of t minus MTTF square.

So, once we put it then this will become as we know from the formula this is integration from t squared FTDT minus mean square, mean here is MTTF and if we integrate this we get the 2MTTF square. So 2MTTF square minus MTTF square gives us MTTF square, and we know what is MTTF that is 1 upon lambda, so this becomes 1 upon lambda square.

$$\sigma^2 = \int_0^\infty t^2 f(t) dt - MTTF^2 = MTTF^2 = \frac{1}{\lambda^2}$$

So standard deviation for this is square root of variance, that is will become one upon lambda. So if we see here that our standard deviation for exponential distribution is same as MTTF. So, if our

MTTF is increasing, that means variability in the distribution is also increasing that means uncertainty in our distribution is also increasing.

$$\sigma = \frac{1}{\lambda} = \text{MTTF}$$

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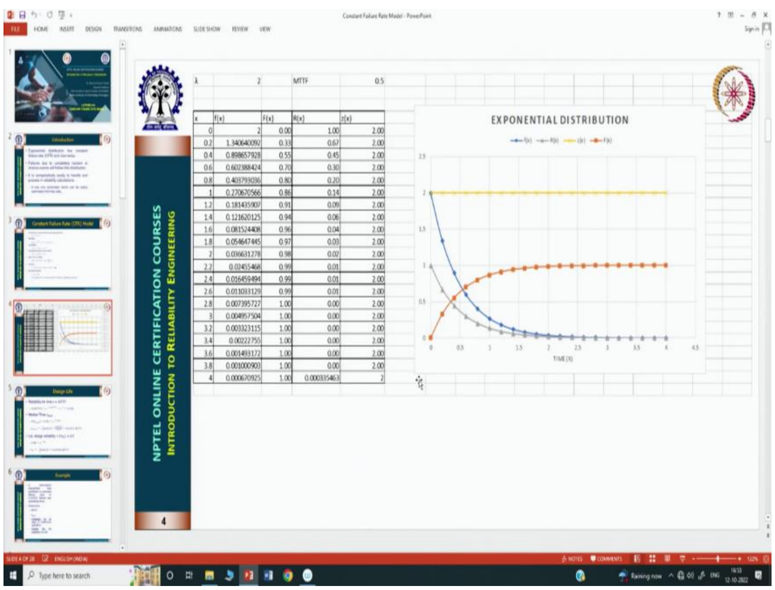

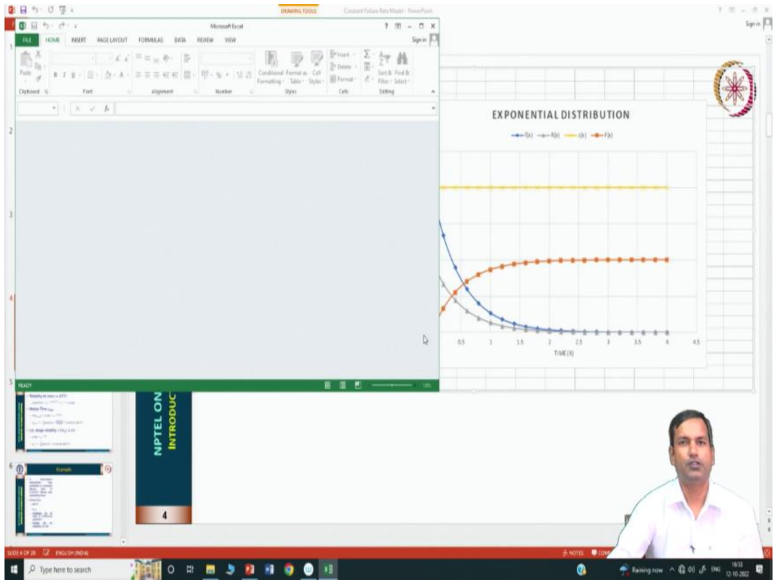
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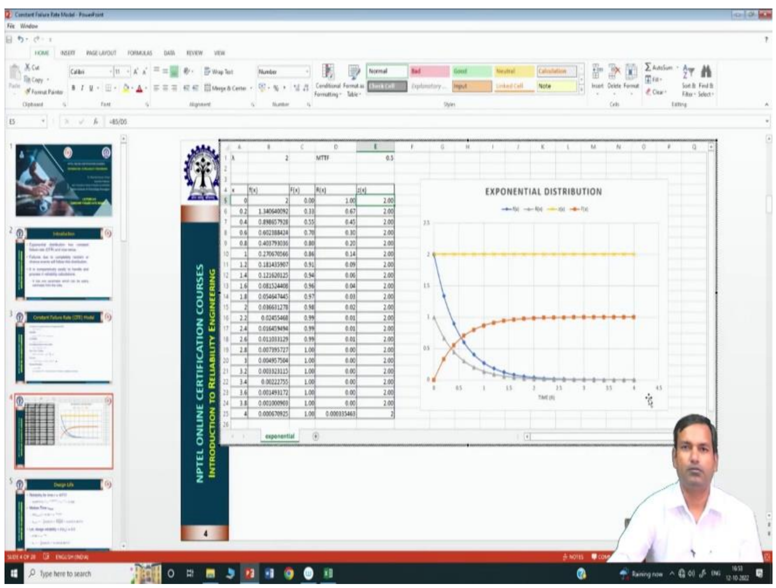
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λ 2 MTTF 0.5
x f(x) F(x) R(x) z(x)
0 2 0.00 1.00 2.00
0.2 1.340640092 0.33 0.67 2.00
0.4 0.898657928 0.55 0.45 2.00
0.6 0.602388424 0.70 0.30 2.00
0.8 0.403793036 0.80 0.20 2.00
1 0.270670566 0.86 0.14 2.00
1.2 0.181435907 0.91 0.09 2.00
1.4 0.121620125 0.94 0.06 2.00
1.6 0.081524408 0.96 0.04 2.00
1.8 0.054647445 0.97 0.03 2.00
2 0.036631278 0.98 0.02 2.00
2.2 0.02455468 0.99 0.01 2.00
2.4 0.016459494 0.99 0.01 2.00
2.6 0.011033129 0.99 0.01 2.00
2.8 0.007395727 1.00 0.00 2.00
3 0.004957504 1.00 0.00 2.00
3.2 0.003323115 1.00 0.00 2.00
3.4 0.00222755 1.00 0.00 2.00
3.6 0.001493172 1.00 0.00 2.00
3.8 0.001000903 1.00 0.00 2.00
4 0.000670925 1.00 0.000335463 2
exponential
EXPONENTIAL DISTRIBUTION

x	f(x)	F(x)	R(x)	z(x)
0	2	0.00	1.00	2.00
0.2	1.340640092	0.33	0.67	2.00
0.4	0.898657928	0.55	0.45	2.00
0.6	0.602388424	0.70	0.30	2.00
0.8	0.403793036	0.80	0.20	2.00
1	0.270670566	0.86	0.14	2.00
1.2	0.181435907	0.91	0.09	2.00
1.4	0.121620125	0.94	0.06	2.00
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2.2	0.02455468	0.99	0.01	2.00
2.4	0.016459494	0.99	0.01	2.00
2.6	0.011033129	0.99	0.01	2.00
2.8	0.007395727	1.00	0.00	2.00
3	0.004957504	1.00	0.00	2.00
3.2	0.003323115	1.00	0.00	2.00
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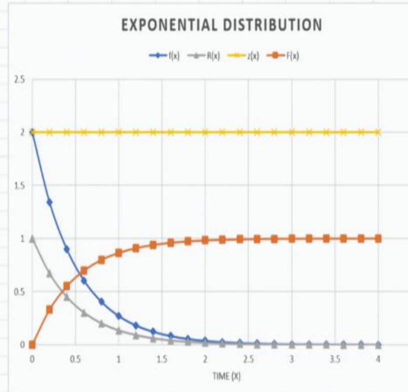
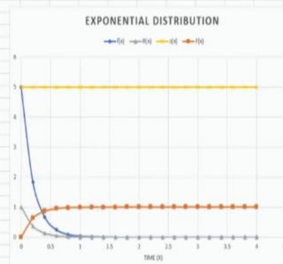
3.8	0.001000903	1.00	0.00	2.00
4	0.000670925	1.00	0.000335463	2



λ	5		MTTF	0.2

x	f(x)	F(x)	R(x)	z(x)
0	5	0.00	1.00	5.00
0.2	1.839397206	0.63	0.37	5.00
0.4	0.676676416	0.86	0.14	5.00
0.6	0.248935342	0.95	0.05	5.00
0.8	0.091578194	0.98	0.02	5.00
1	0.033689735	0.99	0.01	5.00
1.2	0.012393761	1.00	0.00	5.00
1.4	0.00455941	1.00	0.00	5.00
1.6	0.001677313	1.00	0.00	5.00
1.8	0.000617049	1.00	0.00	5.00
2	0.000227	1.00	0.00	5.00
2.2	8.35085E-05	1.00	0.00	5.00
2.4	3.07211E-05	1.00	0.00	5.00
2.6	1.13016E-05	1.00	0.00	5.00
2.8	4.15764E-06	1.00	0.00	5.00
3	1.52951E-06	1.00	0.00	5.00
3.2	5.62676E-07	1.00	0.00	5.00
3.4	2.06997E-07	1.00	0.00	5.00
3.6	7.61499E-08	1.00	0.00	5.00
3.8	2.8014E-08	1.00	0.00	5.00
4	1.03058E-08	1.00	2.06115E-09	5.0000001



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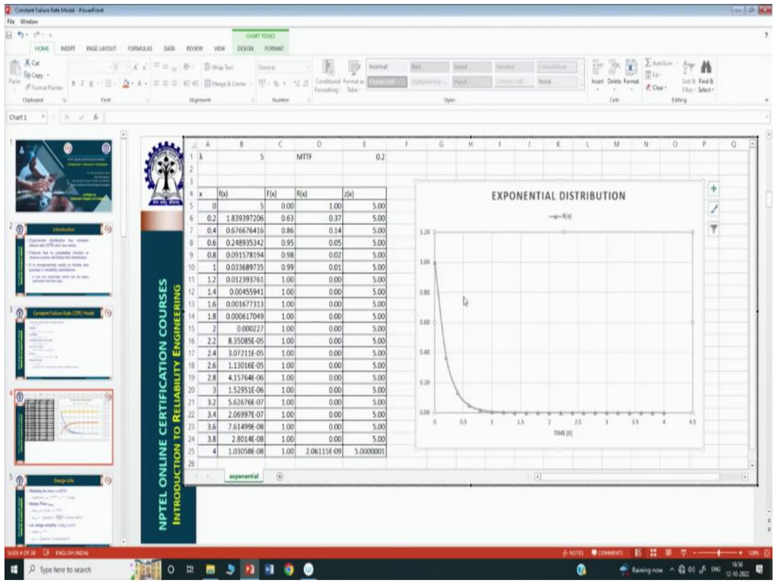

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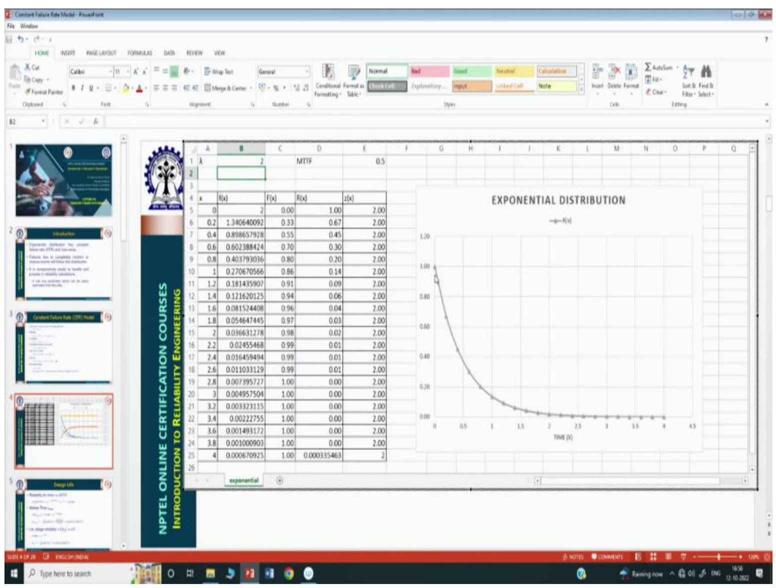

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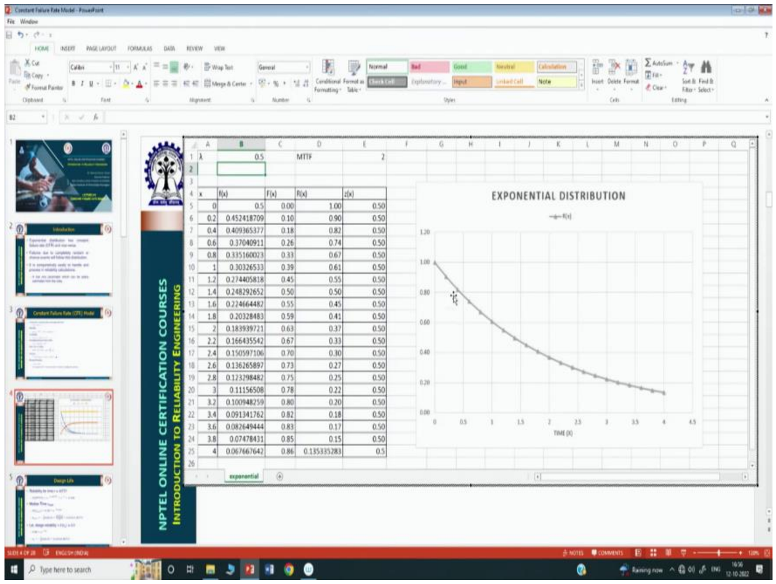
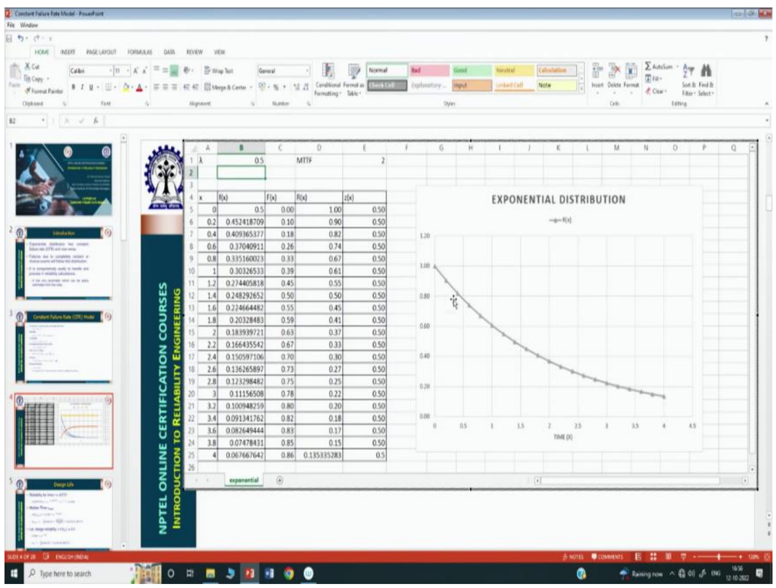


λ	0.5		MTTF	2
x	f(x)	F(x)	R(x)	z(x)
0	0.5	0.00	1.00	0.50
0.2	0.452418709	0.10	0.90	0.50
0.4	0.409365377	0.18	0.82	0.50
0.6	0.37040911	0.26	0.74	0.50
0.8	0.335160023	0.33	0.67	0.50
1	0.30326533	0.39	0.61	0.50
1.2	0.274405818	0.45	0.55	0.50
1.4	0.248292652	0.50	0.50	0.50
1.6	0.224664482	0.55	0.45	0.50
1.8	0.20328483	0.59	0.41	0.50
2	0.183939721	0.63	0.37	0.50
2.2	0.166435542	0.67	0.33	0.50
2.4	0.150597106	0.70	0.30	0.50
2.6	0.136265897	0.73	0.27	0.50
2.8	0.123298482	0.75	0.25	0.50
3	0.11156508	0.78	0.22	0.50
3.2	0.100948259	0.80	0.20	0.50
3.4	0.091341762	0.82	0.18	0.50
3.6	0.082649444	0.83	0.17	0.50
3.8	0.07478431	0.85	0.15	0.50
4	0.067667642	0.86	0.135335283	0.5



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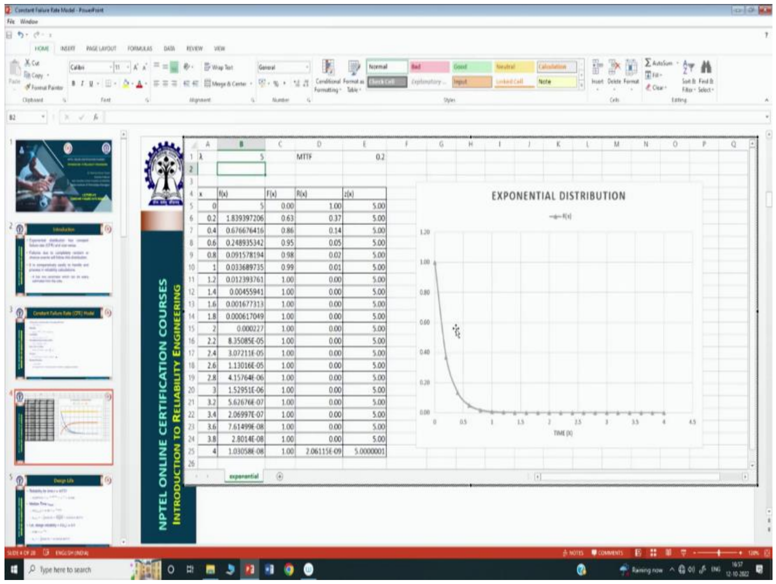

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2.6	1.13016E-05	1.00	0.00	5.00
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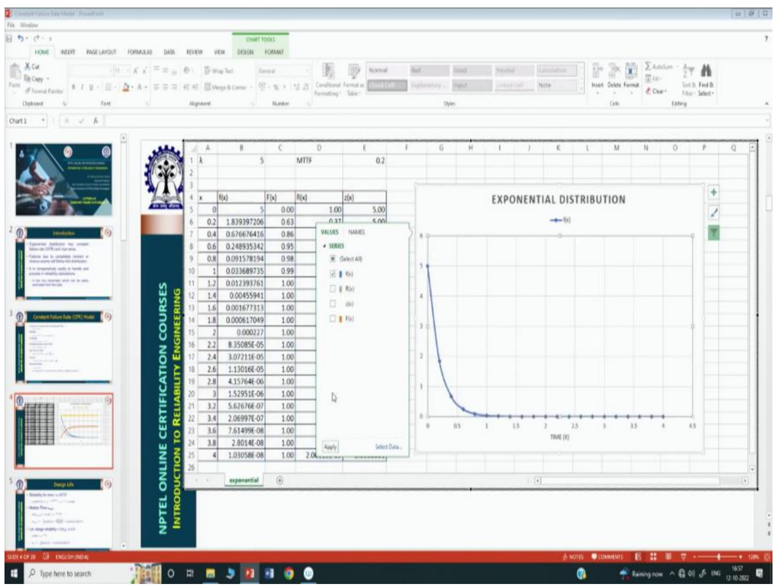
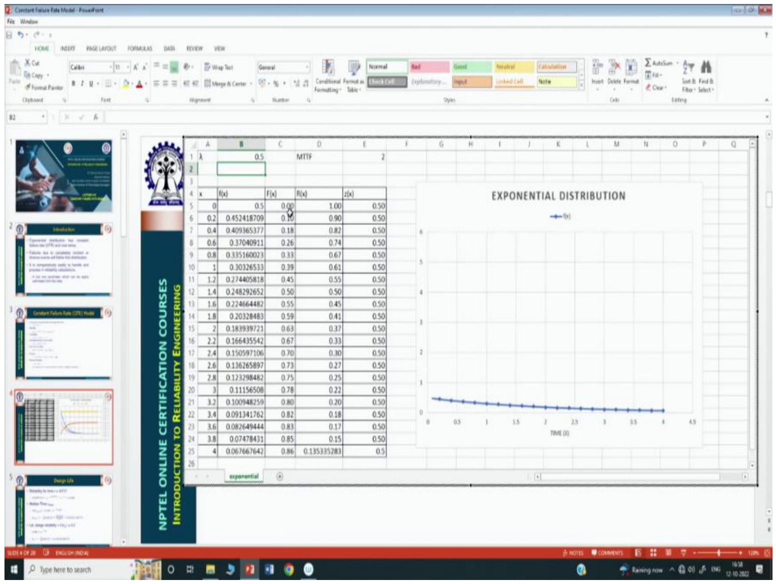
4	1.03058E-08	1.00	2.06115E-09	5.0000001

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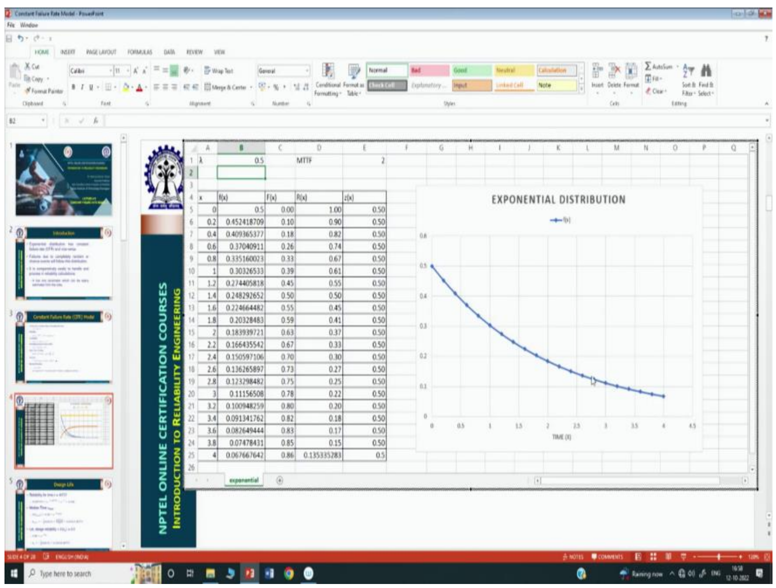
λ 0.5 MTTF 2
x | f(x) | F(x) | R(x) | z(x)
0 | 0.5 | 0.00 | 1.00 | 0.50
0.2 | 0.452418709 | | 0.90 | 0.50
0.4 | 0.409365377 | 0.18 | 0.82 | 0.50
0.6 | 0.37040911 | 0.26 | 0.74 | 0.50
0.8 | 0.335160023 | 0.33 | 0.67 | 0.50
1 | 0.30326533 | 0.39 | 0.61 | 0.50
1.2 | 0.274405818 | 0.45 | 0.55 | 0.50
1.4 | 0.248292652 | 0.50 | 0.50 | 0.50
1.6 | 0.224664482 | 0.55 | 0.45 | 0.50
1.8 | 0.20328483 | 0.59 | 0.41 | 0.50
2 | 0.183939721 | 0.63 | 0.37 | 0.50
2.2 | 0.166435542 | 0.67 | 0.33 | 0.50
2.4 | 0.150597106 | 0.70 | 0.30 | 0.50
2.6 | 0.136265897 | 0.73 | 0.27 | 0.50
2.8 | 0.123298482 | 0.75 | 0.25 | 0.50
3 | 0.11156508 | 0.78 | 0.22 | 0.50
3.2 | 0.100948259 | 0.80 | 0.20 | 0.50
3.4 | 0.091341762 | 0.82 | 0.18 | 0.50
3.6 | 0.082649444 | 0.83 | 0.17 | 0.50
3.8 | 0.07478431 | 0.85 | 0.15 | 0.50
4 | 0.067667642 | 0.86 | 0.135335283 | 0.5
exponential
EXPONENTIAL DISTRIBUTION
f(x)
TIME (X)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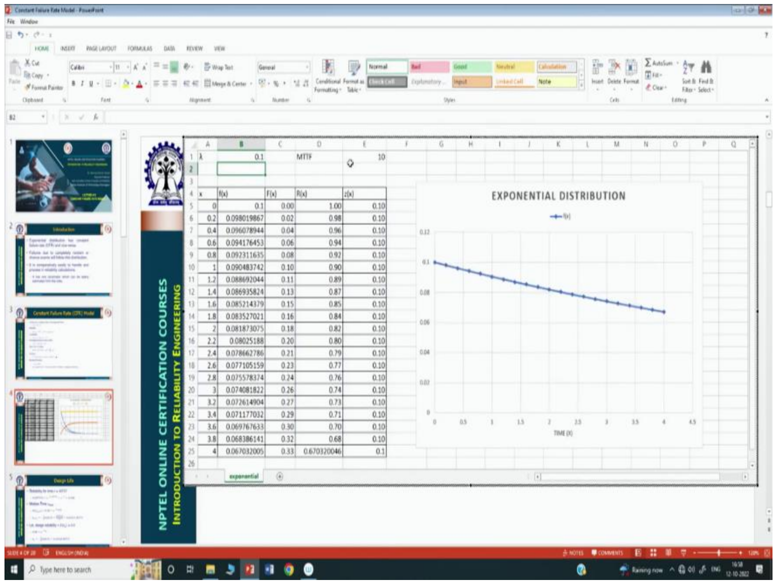
λ 0.5 MTTF 2
x | f(x) | F(x) | R(x) | z(x)
0 | 0.5 | 0.00 | 1.00 | 0.50
0.2 | 0.452418709 | 0.10 | 0.90 | 0.50
0.4 | 0.409365377 | 0.18 | 0.82 | 0.50
0.6 | 0.37040911 | 0.26 | 0.74 | 0.50
0.8 | 0.335160023 | 0.33 | 0.67 | 0.50
1 | 0.30326533 | 0.39 | 0.61 | 0.50
1.2 | 0.274405818 | 0.45 | 0.55 | 0.50
1.4 | 0.248292652 | 0.50 | 0.50 | 0.50
1.6 | 0.224664482 | 0.55 | 0.45 | 0.50
1.8 | 0.20328483 | 0.59 | 0.41 | 0.50
2 | 0.183939721 | 0.63 | 0.37 | 0.50
2.2 | 0.166435542 | 0.67 | 0.33 | 0.50
2.4 | 0.150597106 | 0.70 | 0.30 | 0.50
2.6 | 0.136265897 | 0.73 | 0.27 | 0.50
2.8 | 0.123298482 | 0.75 | 0.25 | 0.50
3 | 0.11156508 | 0.78 | 0.22 | 0.50
3.2 | 0.100948259 | 0.80 | 0.20 | 0.50
3.4 | 0.091341762 | 0.82 | 0.18 | 0.50
3.6 | 0.082649444 | 0.83 | 0.17 | 0.50
3.8 | 0.07478431 | 0.85 | 0.15 | 0.50
4 | 0.067667642 | 0.86 | 0.135335283 | 0.5
exponential
EXPONENTIAL DISTRIBUTION
f(x)
TIME (X)


EXPONENTIAL DISTRIBUTION
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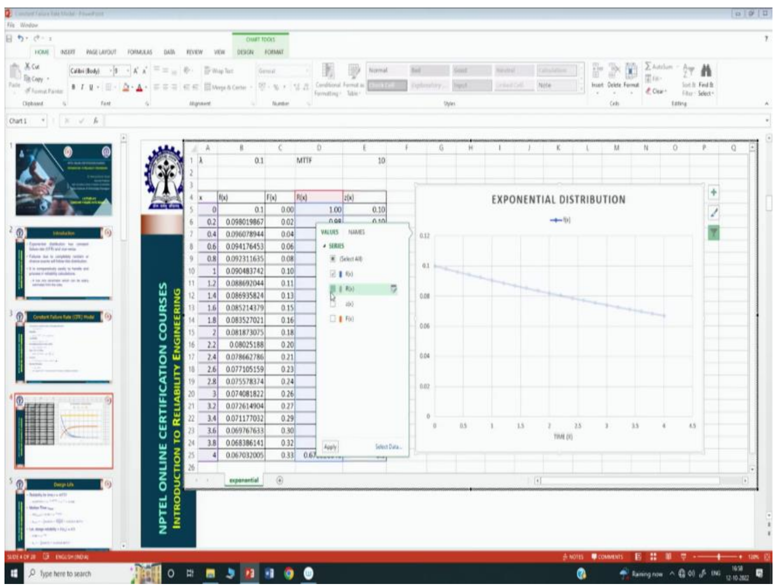
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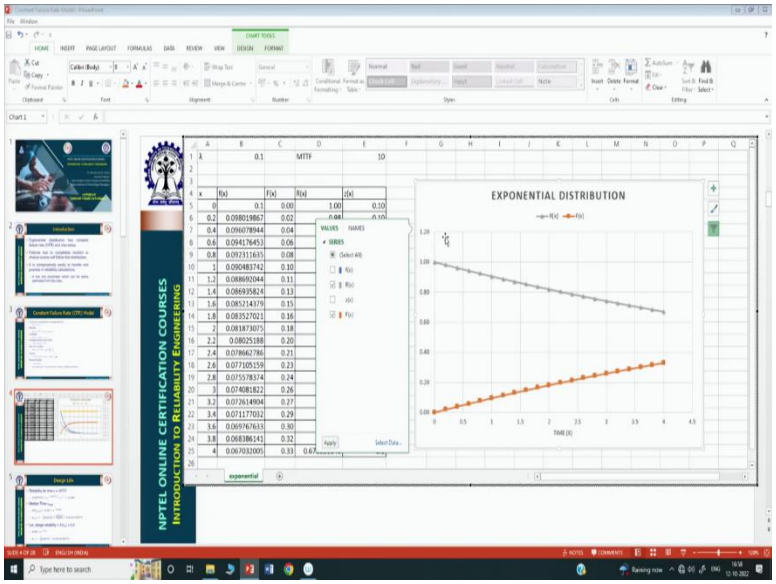
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INTRODUCTION TO RELIABILITY ENGINEERING
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x f(x) F(x) R(x) z(x)
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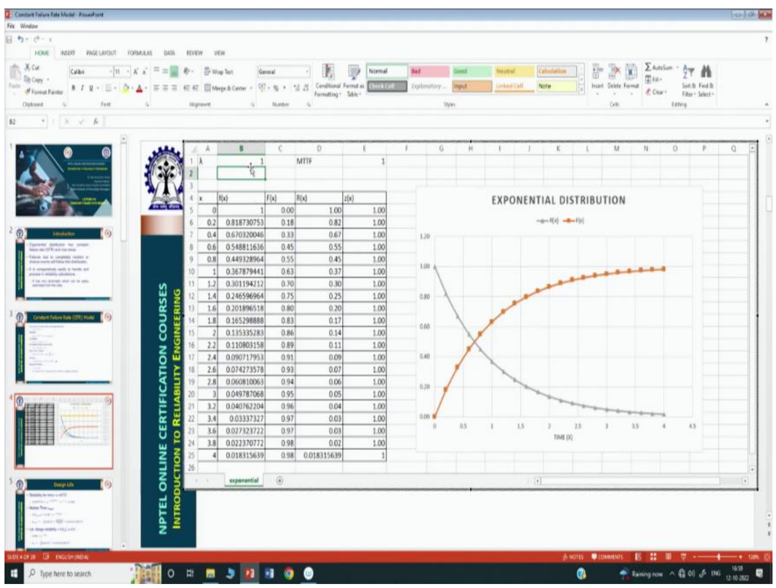
0.2 0.098019867 0.02
0.4 0.096078944 0.04
0.6 0.094176453 0.06
0.8 0.092311635 0.08
1 0.090483742 0.10
1.2 0.088692044 0.11
1.4 0.086935824 0.13
1.6 0.085214379 0.15
1.8 0.083527021 0.16
2 0.081873075 0.18
2.2 0.08025188 0.20
2.4 0.078662786 0.21
2.6 0.077105159 0.23
2.8 0.075578374 0.24
3 0.074081822 0.26
3.2 0.072614904 0.27
3.4 0.071177032 0.29
3.6 0.069767633 0.30
3.8 0.068386141 0.32
4 0.067032005 0.33
VALUES NAMES
SERIES
(Select All)
f(x)
R(x)
z(x)
F(x)
Apply
Select Data...
exponential
EXPONENTIAL DISTRIBUTION
R(x) F(x)
TIME (X)


NPTEL ONLINE CERTIFICATION COURSES
INTRODUCTION TO RELIABILITY ENGINEERING
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x f(x) F(x) R(x) z(x)
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0.2 0.818730753 0.18 0.82 1.00
0.4 0.670320046 0.33 0.67 1.00
0.6 0.548811636 0.45 0.55 1.00
0.8 0.449328964 0.55 0.45 1.00
1 0.367879441 0.63 0.37 1.00
1.2 0.301194212 0.70 0.30 1.00
1.4 0.246596964 0.75 0.25 1.00
1.6 0.201896518 0.80 0.20 1.00
1.8 0.165298888 0.83 0.17 1.00
2 0.135335283 0.86 0.14 1.00
2.2 0.110803158 0.89 0.11 1.00
2.4 0.090717953 0.91 0.09 1.00
2.6 0.074273578 0.93 0.07 1.00
2.8 0.060810063 0.94 0.06 1.00
3 0.049787068 0.95 0.05 1.00
3.2 0.040762204 0.96 0.04 1.00
3.4 0.03337327 0.97 0.03 1.00
3.6 0.027323722 0.97 0.03 1.00
3.8 0.022370772 0.98 0.02 1.00
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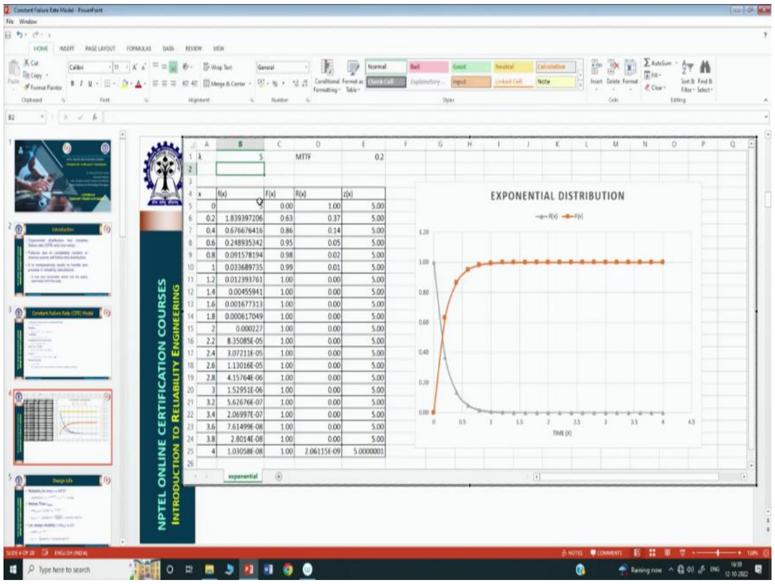
exponential
EXPONENTIAL DISTRIBUTION
R(x) F(x)
TIME (X)



Now, let us discuss how this exponential distribution can be varied over various functions. So, for that, let us see if I think we may not be able to evaluate here I will show it in a, here, so, let us see in this view, if I have just put a Excel sheet here that is Excel sheet if we see this Excel sheet is for same, just let me expand this a little bit, if you see it here, I have just put all the formulas here, the only parameter here is, just give me a second lambda. So the parameter lambda now I can visualize that if I am changing value of lambda how various values how this distribution will vary.

I am sorry, just give me a second. So, lambda is here, if you see this exponential distribution, I have plotted lambda here this is my lambda value. So, since lambda is constant, this will always be a straight line, it does not change with time.

Next, this value is small effects that is probability density function. So probability density function, if you see it starts with high value and then decreases the starting value of this is same as the lambda value that is 2, because at the start, as we will see, if we put t equal to 0, then lambda e to the power minus lambda t when we put t equal to 0 it will become lambda, it starts from lambda, then it will keep on decreasing in an exponential rate.

Then we have this value as the reliability, reliability always start from one then keep on exponentially decreasing. Now, similarly, I have also plotted the unreliability, unreliability always start from 0 then keeps on increasing. So, this value I have plotted for lambda value equal to 2 and this I plotted on a time scale from 0 to 4. Now, if I let us say change this from 2 to 5, then let us see what kind of change happens.

When I change to 5 if you see that my this fall has become faster, the FT which was slowly changing now is changing very fast it is coming down very fast. Similarly, reliability is also coming down very fast. Let us for more understanding let us do one by one let us see how it is affecting the reliability first. Let us see how reliability is affected here.

So, if I put lambda equal to 2, lambda equal to 2 means, my MTTF is 0.5 hour in that case this will look like this, if I put lambda equal to let us say 0.5, you see it is almost looking like a it is taking larger time to decrease because MTTF is higher, if I put let us say 0.1 it is becoming looking like it is this starting period is looking like almost like a straight line because and decreases only up to 0.6 around.

So, it will be going slowly, slowly sloping over a long range, MTTF is 10 here, but when I put this 5 here, you will see there is a sudden and sharp like within 0.5 hours of 1 hour the reliability becomes almost equal to 0. Similarly, if you look at the impact of changing this parameter on values of FX, FX is our failure density quality density function since quality density functions when lambda is large, what will happen same, because this will follow almost similar pattern as the reliability curve, like 5 is there it is starting from 5 then steeply decreasing that means most of the failure we are observing in this region.

If I make it 0.5, then it is starting from 0.5 but it is taking longer to decrease and it is also not reaching to 0 very fast. If I make it further less 0.1 will see that it is even decreasing small, almost linearly it is looking at because it is the initial degradation only, initial downfall only. Similarly, whatever we have observed RT same values we can observe for if you look FX and RX they are one minus of that. So, whatever the pattern is there opposite pattern will be there for the reliability as you see similar this is decreasing whatever rate it is decreasing it will increase with the same rate.

If I put lambda equal to 1, then it will look like this. If I put lambda equal to 5, it will steeply change quickly reaches to 1 quickly reaches to 0. So, this gives us an idea that how exponential curve will look like and failure densities failure it is always going to be same here. So we will continue our discussion, now here or if we look at it.

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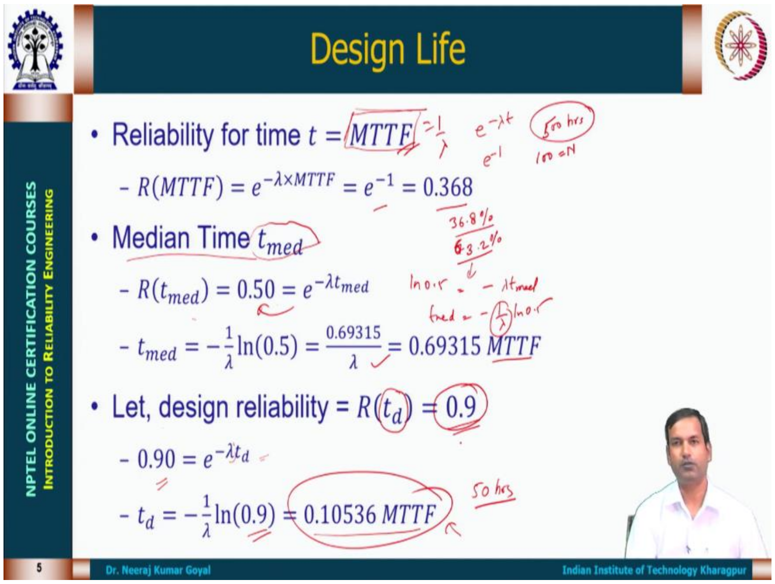

Now let us say many times we are interested to evaluate design life. So reliability for time. So for design life or some other like mean life median life etcetera. We have calculated values for MTTF, which is equal to 1 upon lambda. Now, we want to calculate reliability, how much is the reliability for when time is equal to MTTF or time is equal to 1 upon lambda.

So, we know reliability at time t is e to the power minus lambda t when t is equal to 1 upon lambda this will become e to the power minus 1 and e to the power minus 1 is equal to 0.368. What does it mean that when time is equal to mean time at the time we have almost 36.8 percent failure and how much is only working that means around 73.2 Sorry 63.2. So, we have only 63.2 percent equipment which are working, that what does it mean? That if I have 100 devices which are put on uses, then by MTTF I am time t equal to MTTF if let us say MTTF is 100 hours or let us say 500 hours and I put 100 devices on test.

$$R(MTTF) = e^{-\lambda \times MTTF} = e^{-1} = 0.368$$

Then, in 500 hours I am expecting that out of 100, almost 63 devices will fail and only 37 will be working. Does that means more most of the devices are going to fail. So when we use MTTF as a criteria for reliability, we have to be a little careful. We have also seen in one example earlier that when we have used two different distributions, then even for same MTTF reliability was different.

Reliability is telling us that what is the probability of failure or how much proportion of failure I am expecting from the population, which is much more important parameter to know compared to the MTTF, MTTF can, because of large variability in exponential distribution. MTTF can be a little bit misguiding here, because in case of here, the by the time we say e is equal to MTTF, or many devices proportion of devices have already failed.

So we have to be cautious when using MTTF as the reliability parameter, and MTTF should not be considered that this is a good time for which system because in general, we may like that most of our devices should be working so MTTF is not like always 50 percent, 50 percent if you are interested, then the we go for the median time, median time is by the time t median we will observe 50 percent will be working 50 percent will be failed. So, it is the middle portion of the number of failures.

$$R(t_{med}) = 0.50 = e^{-\lambda t_{med}}$$
$$t_{med} = -\frac{1}{\lambda}\ln(0.5) = \frac{0.69315}{\lambda} = 0.69315\ \text{MTTF}$$
$$R(t_d) = 0.9$$
$$0.90 = e^{-\lambda t_d}$$
$$t_d = -\frac{1}{\lambda}\ln(0.9) = 0.10536 MTTF$$

So, by the time we expect that 50 percent of the failures, so, reliability becomes 0.5 and that is equal to e to the power minus time t equal to t median. So, using this reverse calculation, we can get it if I take log both side, so ln of 0.5 will be equal to minus lambda t median. So, t median will be equal to minus 1 upon lambda ln of 0.5 same thing here and which comes out to be this value.

So, if you see that one upon lambda is coming now, 1 upon lambda we can say it is equal to MTTF, so, that can be replaced as MTTF. So, if you see that median time is around 0.69 or around 70 percent of the MTTF. So, median is less than MTTF. Let us say we are interested to know the time by which our some reliability target will be achieved. So, we are producing certain product let us if we want to say our reliability target is 0.9, we are promising that our reliability that means 0.9 reliability will be there for a certain time I want to know that how much will be that time.

So, that means, we are promising that our design will have only 10 percent failure by time td. So, this becomes our design life, td becomes our design life, that in design life we are expecting only 90 percent fail. So, we are designing for 90 percent reliability 10 percent failures only. So, reliability e to the power minus lambda into td will become 0.9 here, you can also design for 95 percent reliability, in that case this will become 0.95, you can also design for 98 percent reliability.

So, reliability how much reliability you are designing for that is to be determined by the based on the customer or based on the product, if we have a product requirement which requires a very high reliability to be delivered, then accordingly we can set this reliability and same value can be used here and according to that, if we have the failure rate information constant failure rate parameter is known to us then we can get the value of TD. So, in this case TD will be equal to same as we have done earlier minus 1 upon lambda ln of 0.9 and that will come out to be 0.1, this I have calculated and put it here that is around 10 percent of MTTF.

So that means whatever MTTF I have calculated let us say my MTTF was 500 hours. So that means for 500 hours, I was expecting 63 percent failure. But for 10 percent failure, I have to promise the life which is almost like a 10 percent, almost like a 50 hours.

So, for design life, which I will be aiming here will be 50 hours. That means I will be promising customer I will be telling the customer that for 50 hours I am giving you the equipment because in this duration only. I am expecting that I will meet your criteria of reliability of 0.9, though MTTF is 500. So we have to be cautious here it is better to design or promise reliability based on the design reliability for a given reliability value rather than MTTF.

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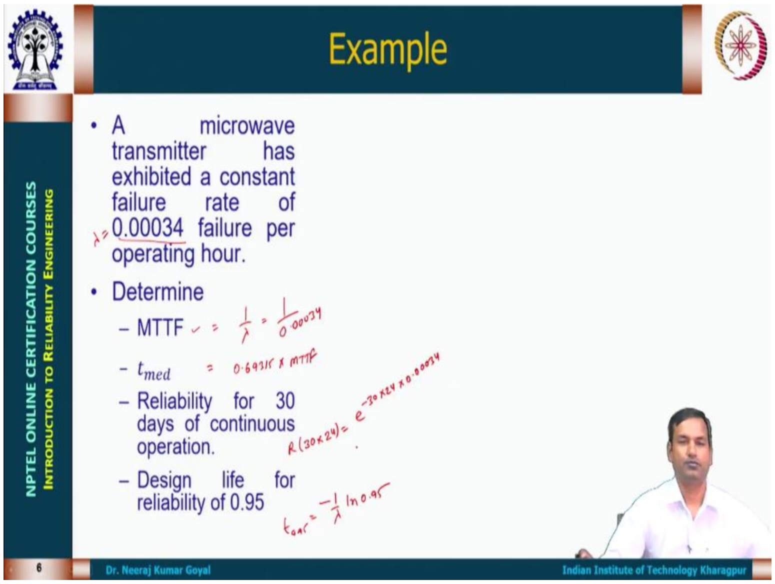

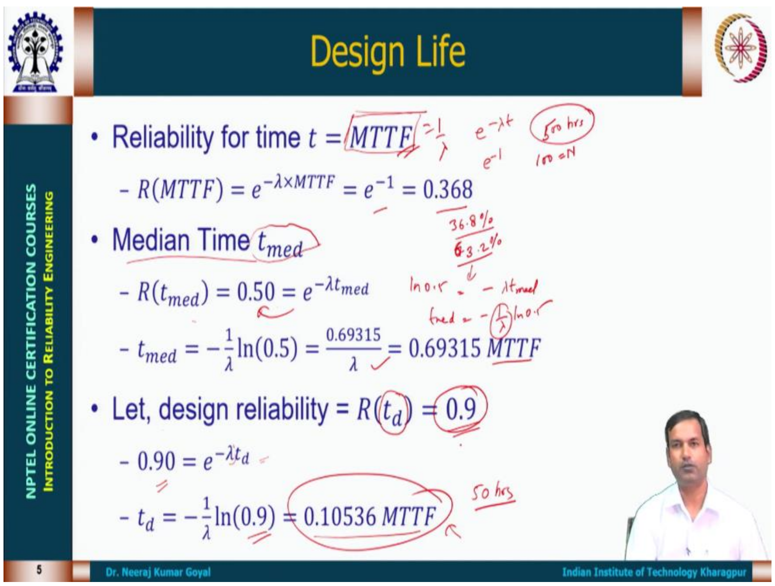

Let us take one example that there is a microwave transmitter and this microwave transmitter is having a failure rate of 0.00034 failures per operating hour. So, can we determine MTTF we know MTTF. So, this becomes our lambda say MTTF will be equal to 1 upon lambda that will be equal to 1 upon 0.00034, we want to know t median, t median as we have seen earlier that is 69315 or around 70 percent 0.69315 into whatever we have got here that is MTTF we can calculate this.

Similarly, if we want to know reliability for 30 days of continuous operation now, we want to know reliability of 30 days, but this is given in hour. So, we have to convert days into hours, so, 30 into 24 that will become number of hours, this will be equal to, e to the power minus 30 into 24 multiply by lambda lambda is 0.00034, design life for reliability of 0.95 so, that is same minus 1 upon lambda ln of 0.95 we calculate this we will get the t 0.95.

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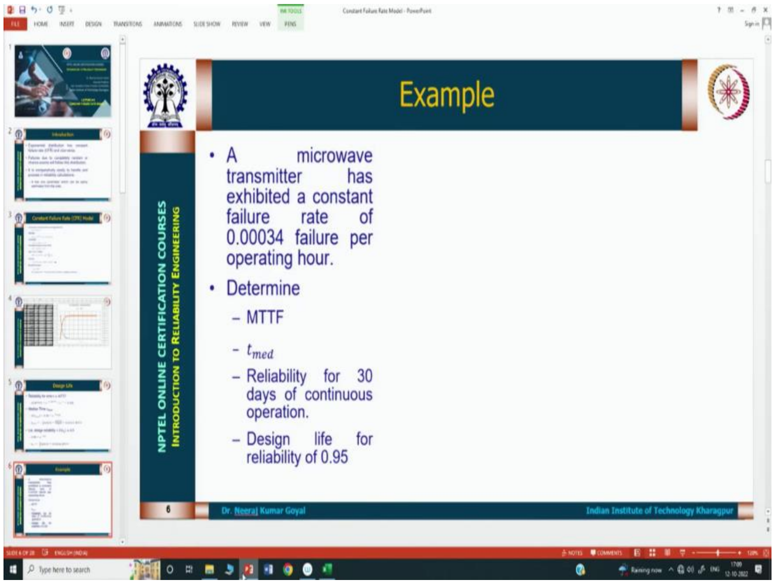

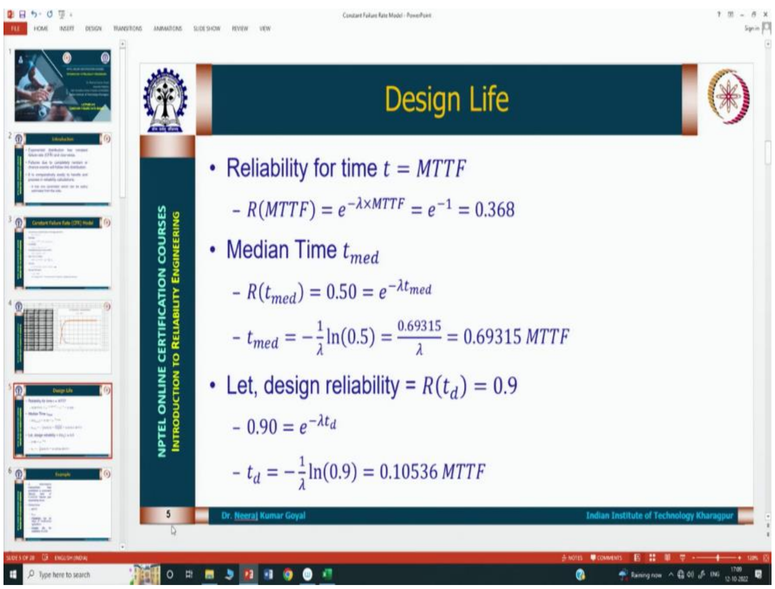

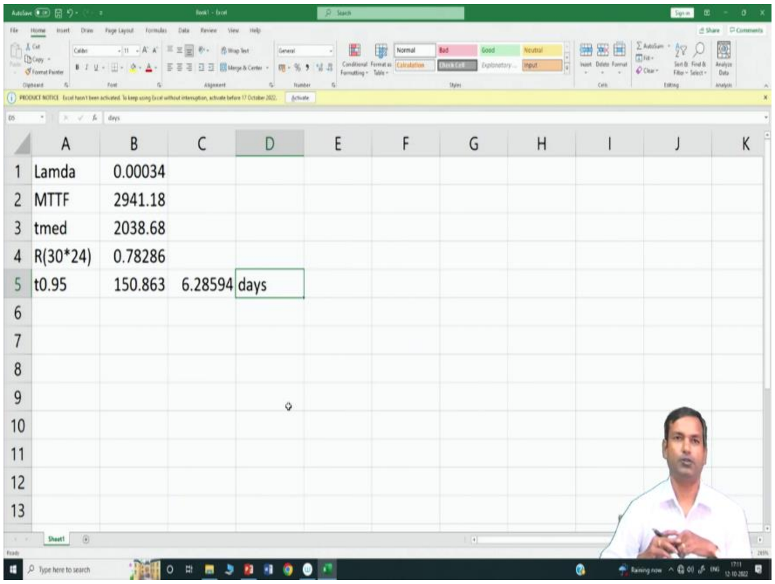

So, here are these calculations we can do if it is not shown here so, let us see if we do it here. So, we have seen the formula if we want we can show it here by using the Excel sheet. So, I think this is being used first time. So, my lambda here was I will just copy that same thing from here, that was 0.00034, I want to calculate MTTF. So, MTTF is equal to this is equal to 1 divided by lambda, then we can calculate t median, median time to failure is equal to MTTF into 0.69315 I think, 315.

Next we have to calculate is reliability for 30 days are 30 into 24 so, this is equal to exponential minus lambda into 30 into 24, our reliability comes out to be 78 percent or 0.78. Next what we wanted to calculate is design life for reliability of 0.95 t 0.95. As we have seen this is equal to minus ln of 0.95 divided by lambda, lambda is this.

So my design life comes out to be around 150 hours. If we want to convert into days, I can divide this by 24, because it is having the continuous operation, so around 6 days, so, around 6 days, there will be that design life for this equipment, I will be promising the customer that for 6 days you, it will continuously run without any much problem, reliability will be 0.95. So, we will stop it here and we will continue our discussion in the next lecture. Thank you.