Introduction to Reliability Engineering
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Lecture 08
Constant Failure Rate Model- III

Hello everyone, we will continue our discussion from our previous lecture that is on constant failure rate model or exponential distribution.

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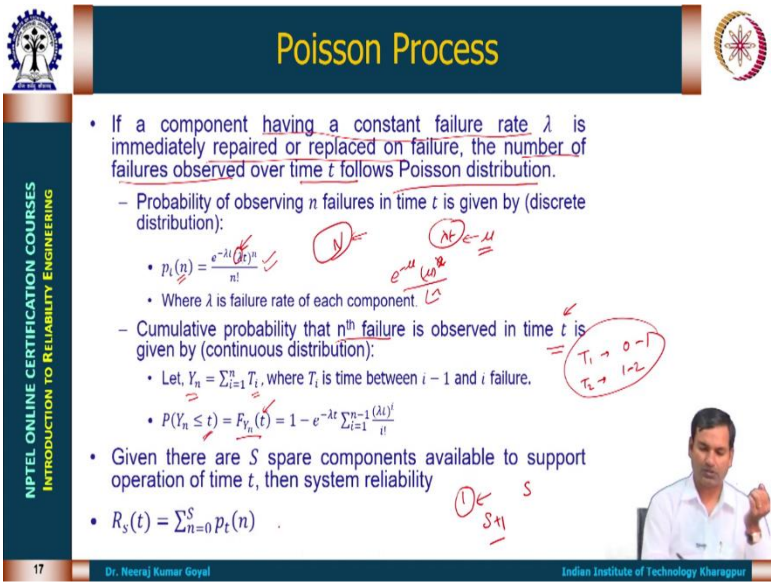

We have already studied about exponential distribution, how to get failure rate, how to get a PDF, reliability, unreliability etcetera and then we discuss that how some in some particular application exponential distribution can be used to model the situation. Now, we are going to discuss another situation in which let us say again, if we have one component which is failing the constant failure rate or the exponential distribution.

$$p_t(n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

So, and now, what happens this component if it fails, then it gets immediately repaired or replaced. So, since there is an immediate repair and replace we do not lose much time between the replacement or repair and so, what happens here one equipment on failure gets replaced so, our function is not lost or we are able to continuously use the system. So, here what will happen we will observe certain number of failures in a given time. So, that number of failures depends on the distribution which you are using.

Now, this number of failures if we take, number of failures we are writing as let us say n, number of failures n is a random variable which is discrete random variable that is numbers 0, 1, 2, 3, like that. Now, this number of failure, which is a random variable now, because in a given time t there may be 0 failures, there may be 1 failure, 2 failure, 3 failure and their probabilities would be different. So, the probability that number of failures would be a small n amount. So, this very small n can be any number, this probability is given by the Poisson distribution, this Poisson distribution we have already discussed in the discrete distributions.

So, as we discussed earlier that, when we have the constant failure rate or constant arrival rate, arrival rate is not a function of time, that is what we are giving us lambda, lambda is a constant failure rate, then expected number of failure in the small time t will be lambda t because unit time we are expecting lambda failures. So, in time t this will become lambda t expected number of values, this becomes our mu, mu is the parameter of Poisson distribution which is also mean and variance of the Poisson distribution.

So, this gives us the Poisson distribution, Poisson distribution as we earlier discussed this is e to the power minus mu, mu to the power x divided by or mu to the power n in this case divided by factorial n, we are able to get the same thing here. So, once we use this formula, we can get the probability of 0 failure, probability of 1 failure, probability of 2 failure, probability of 3 failure. So, this becomes very easy to solve spare parts related problems.

$$Y_n = \sum_{i=1}^{n} T_i$$

$$P\left(Y_n \le t\right) = F_{Y_n}(t) = 1 - e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^2}{i!}$$

So, similarly, this is in number of failures zone, we can also get the probability in time zone, that means, what is the probability that we will observe n number of failures in time t, so, here time t is the random variable. So, we want to know what is the time to nth failure, so, time to nth failure if you are representing as Y_n. So, time to enter failure Y_n will be submission of T_i, here T_i is the time to each failure or time between failures. So, T1 means, time from 0 to first failure, how much time is spent, T2 means, how much time is spent from 1 to 2 failure first to second failure.

So, this is time it is 0 to 1 failure, this is time between 1 to 2 failure like that, and summation of this will give us the time from 0 to till that failure time. So, if I am saying Y1 that means this is essentially equal to T1 time to first failure. Y2 means time to second failure, which is submission of T1 and T2, Y3 is time to third failure, that is T1 plus T2 plus T3. So, Yn is essentially submission of Ti i equal to 1 two n.

Now we want to know what is this Yn, Yn is time, time to nth failure, what is the probability that time to nth failure is less than t time to failure is nth failure is less than t means our nth failure happens before time t small time t , that means it is unreliability, unreliability when I have the n components in my hand, so, that means all n components are failing before time t.

So that gives me unreliability. So, because I do not have any further spears or I do not have any further components, so, if all the spares and components failed before time t, I will not be able to use the system, it will be failed this we can, this is a cumulative distribution which we can represent as FYn t, t is the random variable.

So, we are representing it here and Yn is the random variable representing the at time t, one this value turns out to be if you see it here this is very similar to this the only thing is this is 1 minus of this that means 1 minus of because it will work what is the reliability here, if I have n component reliability would be there will be there will be 0 failure that will be n equal to 0 sorry, up to 1 failure, 2 failure, 3 failure 4 failure like that, if I am having 4 then it can treat up to there is a failure of 0, 1, 2, 3, 4 like that.

So, here the FYn t is given it 1 minus e to the power minus lambda t and equal to i equal to 0 to n minus 1 lambda t raised to the power i factorial i. So, if we have s squares which are available to us then this would be equal to the system reliability would be equal to all the cases up to S number of, because we have one device which is already working, then we have S, so total devices are S plus 1 here. So, here the value would turn out to be summation that 0 failure up to S failure because S plus 1 failure means total system is lost, we cannot use it. So, this turns out to be system reliability.

$$R_S(t) = \sum_{n=0}^{S} p_t(n)$$

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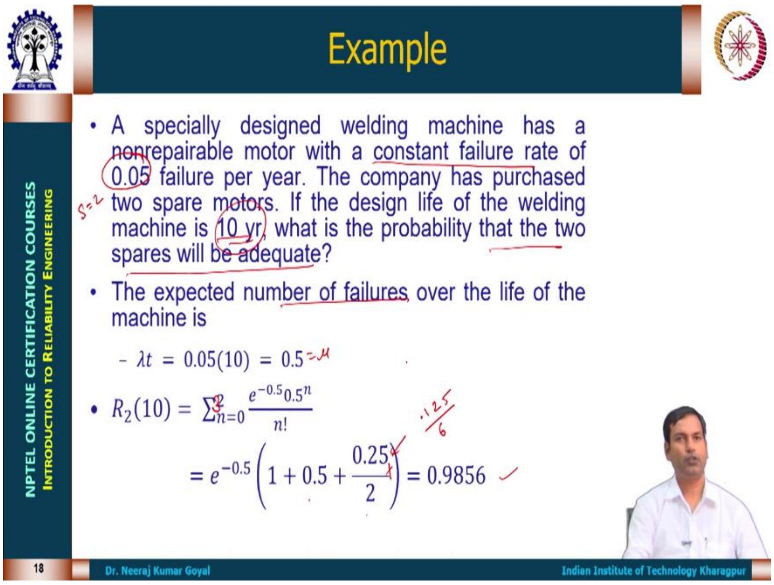

Let us take an example, I specially designed welding machine has a non-repairable motor with a constant failure rate of 0.05 failure per year, the company has purchased 2 spare motors. So, we have S equal to 2 here, that means I can sustain I can continue working even, if there are two failures, if third failure occurs after that, I will not be able to use the system, if the design life of welding machine is 10 years, I want to use this up to 10 years.

So, what is the probability that two spares will be adequate, that means my system will continue to work up to 10 years and my spares will be consumed or not consumed, but my system will not being failed state that means what is the here the expected number of failures or mu as we wrote mu is equal to lambda t.

$$\lambda t = 0.05(10) = 0.5$$

$$R_2(10) = \sum_{n=0}^{2} \frac{e^{-0.5} 0.5^n}{n!}$$

$$= e^{-0.5}\left(1 + 0.5 + \frac{0.25}{2}\right) = 0.9856$$

So, lambda is 0.05 t is 10 years, 0.05 per year and multiply by 10. We have to keep remember the unit is it is per year. So 10 into 0.05, this is the 5 0.5. So, this is our mu, now we can get the reliability, reliability means 0 failure, 1 failure, 2 failure it can sustain and it will be reliable. If third failure occurs, then it will be unreliable. So that R2 10 comes out to be n equal to 0 to 2 e to the power minus 0.5, 0.5 raised to the power n divided by 5 factorial n. So, e to the power 0.5 we can minus 0.5 you can take outside and then for n equal to 0 this will become 0.5 raised to the 0 divided by factorial 0.

So that will be 1 divided by 1 for n equal to 2 this will become 0.5, sorry n equal to 1 it will become 0.5 divided by factorial 1, factorial 1 is 1, so plus 0.5 for n equal to 2 this will become 0.25 squared 0.5 divided by factorial 2 factorial 2 is 2 0.25 by 2, when we solve this we will be able to get 0.9856. So, as we see this has a very interesting application that when we have spares, how can we calculate the reliability of the system for a given time?

Or we can also see that how many we can reversely see that how many spares would be adequate like, this is giving me 0.98, so that means almost 98 0.5 percent reliability I am able to achieve.

If my target is let us say 99 percent, in that case, I can add 1 more spare here, I can then if I add 1 more spare this will become a 3. So, in that case, I will add additional I will get additional reliability here, how much that will be the summation would be added is 0.5 raised to the power 3. So that would be 20, but you found in this Bara 0.125 divided by factorial 3 factorial 3 is 6.

So, we can see whether this is sufficient or not or how many spares are required. So, this helps us to have the reliability estimate in case we, but the condition is that the components should follow the constant failure rate, if it is not constant failure rate then it will not be Poisson process and we will not be able to use this formula.

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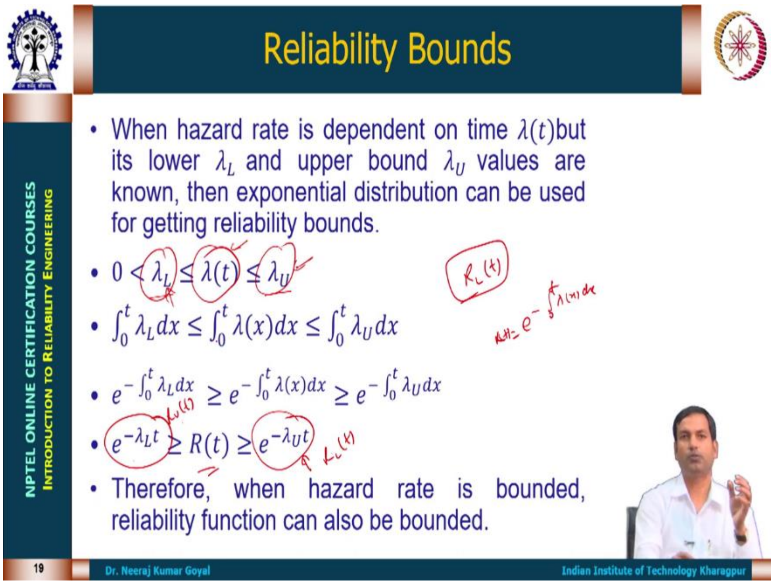

Sometimes we are interested to know reliability bounds, most of the time we are interested in lower reliability bounds we are interested in this value that reliability is greater than something. So, R we RL t, that means at time t my reliability is minimum this better than this is good.

Similarly, for things which are of negative nature, which is undesired, we try to we are interested in upper limit like our failure probability is not more than a certain value, reliability is better than this. So, we always want to have an optimistic conservative estimate which will give us a optimistic view. So, RL gives us a conservative estimate on reliability, we expect a better reality than this. But sometimes, if required, we can also calculate the upper reliability is sorry, this is our RL t this is over RU t.

Now, in case we have the failure rate, failure rate is a function of time or it can be constant failure rate, if it is constant failure rate, we consider this as exponential distribution, let us say failure rate is having a time function also, but we know that at the time t which is of time of interest at time t lambda value is lying somewhere between lambda L and lambda U, that means lambda t value we know this is going to fall between lambda L lower limit on lambda and upper limit of lambda which is a constant value.

So, anytime we evaluate the bound it will be giving us two values lower limit and upper limit. Now, we want to know if we know the lower limit and upper limit on lambda, this is a constant value now. So, we can use the reliability equation of exponential distribution that will give us the bounds on the reliability. So, since we are taking e to the power minus integration of lambda. So, what happens when lambda is more the reliability will be less.

So, since this is a inverse relationship, that is why for lambda L we get the RU for lower the value of lambda we get the upper value of reliability. Similarly, for lower value of upper value of lambda we get the lower limit on reliability.

$$0 < \lambda_L \leq \lambda(t) \leq \lambda_U$$
$$\int_0^t \lambda_L dx \leq \int_0^t \lambda(x) dx \leq \int_0^t \lambda_U dx$$
$$e^{-\int_0^t \lambda_L dx} \geq e^{-\int_0^t \lambda(x) dx} \geq e^{-\int_0^t \lambda_U dx}$$
$$e^{-\lambda_L t} \geq R(t) \geq e^{-\lambda_U t}$$

So, as we see by using the exponential distribution, we can simply take the e to the power minus lambda Lt which gives us the upper limit on upper bound on the reliability and lambda Ut, it gives us the lower bound on reliability and this can help us to know that reliability even if we do not know that failure rate function is the constant failure rate, but we know the bounds we know that it is bounded in certain value.

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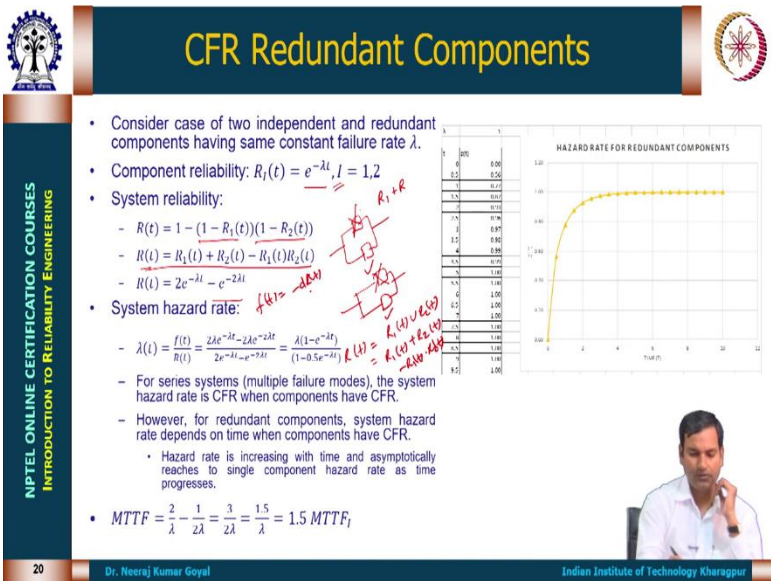

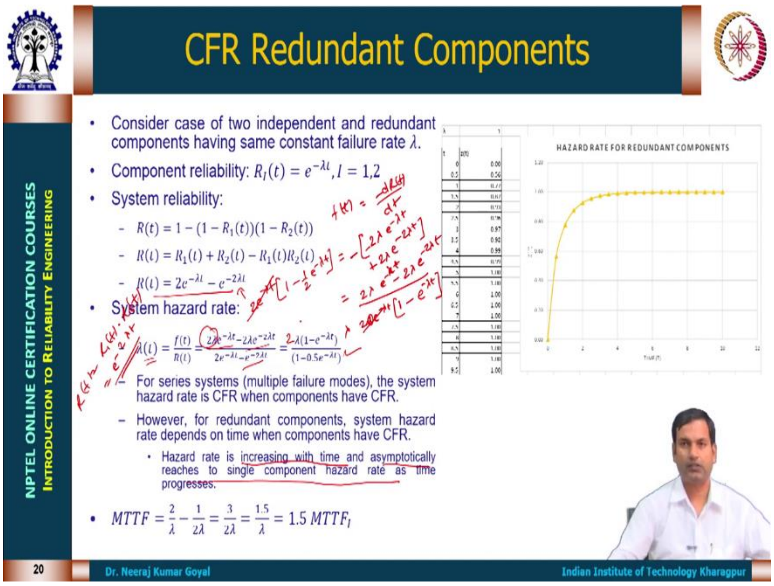

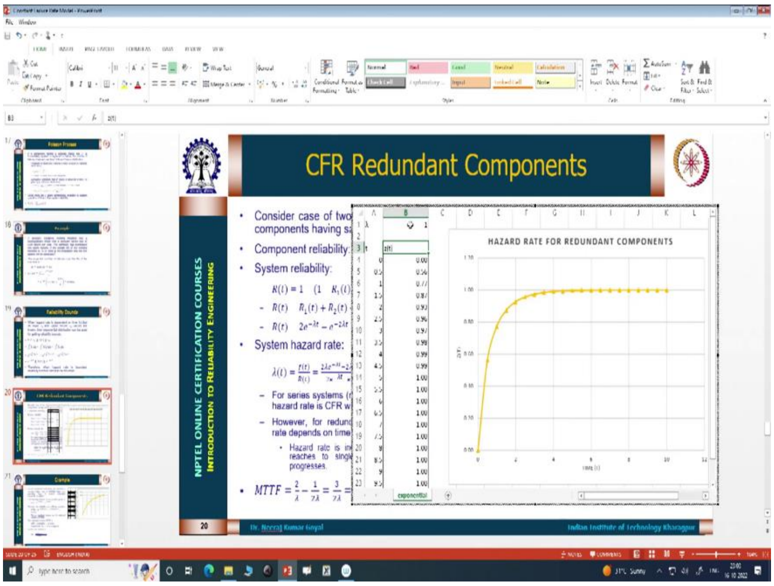
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Component reliability
System reliability
System hazard rate
For series systems hazard rate is CFR
However, for redund rate depends on time
MTTF
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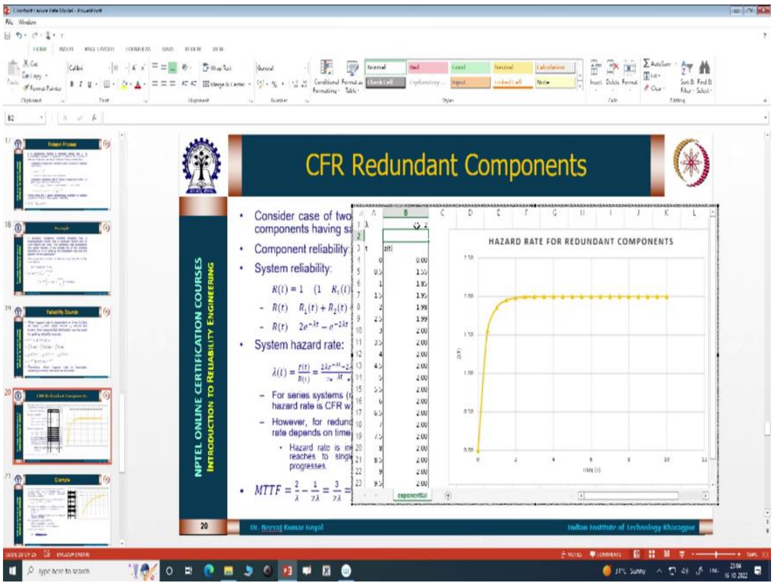
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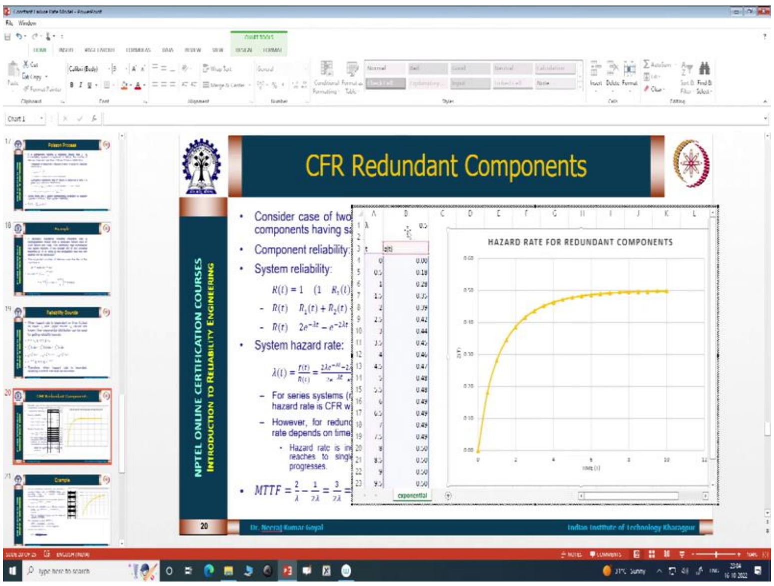
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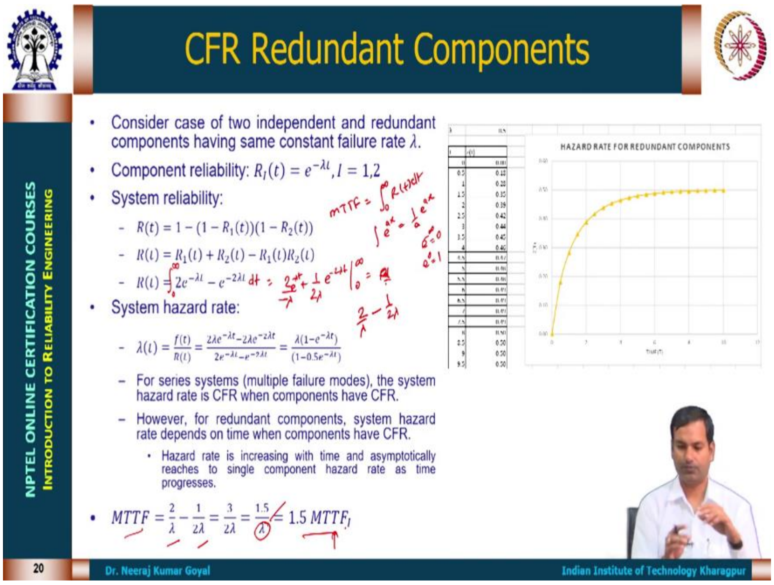

Now, let us see constant failure rate redundant components, redundant components means that we redundant components like we have two components anyone fails even if this fails my system will continue to work. Let us say we have two pipelines are coming to our home from the same source. Now, if one pipeline breaks, then second will continue to work. So, in that case, our job is our water supply continues our water supply does not fail.

So, if we do that, let us say both failure probability of any pipeline is same or any other system similar system there are parallel redundancies is there, then let us say that each component is having a failure rate e to the lambda and reliability is going to be for time t it is going to be e to the power minus lambda t let us say we have two components.

So, R1 t, R2 t we can get. Now, system reliability, we know that for parallel case, parallel cases union case or this will come out to be R1 t plus R2 t, Rt is equal to we can write in two way one is 1 minus multiplication of 1 minus Ri t that is 1 minus R1 t multiplied by 1 minus R2 t, that this we discussed earlier when we discuss the probability.

So, here or we can also get apply the union formula that is Rt is equal to R1 t union with R2 t, or we can say R1. So, this becomes I am generally we do not do this for probability we have to apply union on the events, here the event then we have to call T1 T2, but to simplification for understanding purposes I am writing like this.

So, that becomes R1t plus R2 t minus R1 t into R2 t. So, as you see here, same formula comes out for Rt. Now, if we apply R1 t and R2 t, R1 t is e to the power minus lambda t, R2 t is also e to the power minus lambda t, r so, this becomes 2 into e to the minus lambda t and this is e to the minus lambda t into e to the power minus lambda t. So, that becomes e to the power minus lambda t square or we can say e to the power minus 2 lambda t.

Now, once we have this reliability from the reliability we can get the system hazard rate, hazard rate is given us ft upon Rt where ft is what, ft minus d Rt over dt. So, here if I calculate f t, f t will be equal to minus t Rt over dt.

So, that means Rt is given to me here, I have to differentiate this if I differentiate this, this will become since all is negative. So, when I differentiate this negative of minus differentiating this will be minus 2 into lambda e to the power minus lambda t minus again when I differentiate this will be minus 2 lambda will come out so, minus minus plus 2 lambda e to the power minus 2 lambda t.

So, when I take my negative inside this become 2 lambda e to the power minus lambda t minus 2 lambda e to the power minus 2 lambda t, this becomes our ft same thing we have written here, 2 lambda e to the minus lambda t minus 2 lambda e to the power minus 2 lambda t and what is Rt, Rt is 2 e to the power minus lambda t and e to the four minus 2 lambda t.

Now, here to lambda we can take as common. So, once we take 2 lambda common this will become 2 lambda into e to the power minus lambda t into minus e to the power minus 3, from there again we can take. So, these steps are like this like I will take 2 lambda, 2 lambda e to the minus lambda t common here, first term is taken as common so, this will become 1 minus 2 lambda is taken out e to the power minus lambda t is taken out. So, if I divide this by e to the minus t lambda t the remaining will be e to the power minus lambda t.

Similarly, here if I solve this, if I take 2 e to the power minus lambda t as common, this will become 1 minus Now, here 2 is not there, so, this will become 1 by 2 to the power minus lambda t same thing comes out so, this becomes.

Now, 2 lambda t e to the minus lambda t 2 e to the power minus lambda t can be cancel lambda will be only remaining here lambda will be outside. So, we get lambda into e to the power 1 minus e to the minus lambda t divided by 1 minus 0.05 e to the power minus lambda t. Here if we see this gives us the failure rate.

Now, this failure rate is not constant while if the system was a series system, series system means are both components are required for system to work. In that case Rt would be equal to R1 t multiplied with R2 t in that case this comes out to be e to the power minus 2 lambda t in this case, the system is again constant failure because this is again exponential distribution with parameter 2 lambda.

So, the failure rate, which was lambda earlier has become now 2 lambda, but in case of parallel systems, we are not able to reduce it to the constant value or independent of time as we see this is dependent on time we are not able to subtract it and get it make it to the independent of time. So, here lambda t becomes function of time.

So, for series system failure rate for when all components are failing constant failure rate system also follows a constant failure rate, but for redundant components or parallel systems, the system hazard rate depends on the time, even when components are falling the constant failure rate and this hazarded if you see this hazard rate is increasing with time, this is increasing hazard rate and this asymptotically reaches to those single component hazard rate, finally the hazard rate it will reach to the single component hazard rate as time progresses this we can see with the example, which I have drawn here already as an in an Excel sheet.

So, to see this, I have to open this. So, if you see here, I have taken the value of lambda here. So, when lambda is 1 failure, rate is 1 then I have plotted this for time t equal to 0 to 10 this will look like this. So, if you see it here it is starting for almost 0, because it is time dependent. So, t equal to 0 it is coming like to be 0 almost and then it is increasing, increasing and if you see it is going up to the lambda equal to 1. And if you see this is almost at lambda equal to 4, 4 times of the lambda it is reaching to the, when time t equal to 4 times of MTTF, means 1 upon lambda into 4 around, then it is reaching up to here.

Now, let us say for this case we may not be absolute let now let us say. So, what happens if I increase the lambda failure rate let us say if I make it 2, let us see what happens when failure rate is 2 sorry Num Lock, when failure rate is 2 then if you see this sharper the rise is sharper in a less time it is reaching almost at time t equal to 2 it becomes failure rate equal to 2.

Now, if let us say if I reduce this if I make these 0.5, then what happens it will again rise 0.5, but it will take more time to rise 0.5 that would be around 2 into 4 around 8. If you see in around 8, it reaches to time t equal to 8 hours if failure rate in per hour. The general index use unit use for failure it is per hour per operating hour.

So, if it is not mentioned assume that this is per hour. So, if you see in 8 hour it will be reaching to the failure rate. So, if as we see that larger the failure rate, it will quickly it will try to reach to the hazard rate of single system even though we are having the multiple systems here. So, this helps us to understand that how this hazard rate is changing with time and as we see hazard rate is increasing with time. So, this is increasing hazard rate here. Now for this case, if you want to calculate MTTF, the MTTF turns out to be how we will calculate MTTF, MTTF is as we know, MTTF is integration from 0 to infinity, Rt dt.

So, RT is this. So, I simply have to integrate from 0 to infinity of this, if I integrate what will happen exponent, we know in integration of exponential power leads to the 1 upon whatever is the integration of ea x is 1 upon a e to the power ax. So same thing if we apply, we will get 2 upon minus lambda plus 1 upon 2 lambda, 2 is here, integration of this you will get 1 minus one upon lambda e to the power minus lambda t and this will give 1 upon minus 2 lambda minus minus will become plus and e to the power minus 2 lambda t.

Now, limit 0 to infinity, when we put infinity both will become 0 exponential minus infinity is equal to 0 and exponential 0 will be equal to 1, when we put 0 then this will become 1 and this will be minus minus plus 1, 2 upon lambda that will be equal to 2 upon lambda and this will become minus minus 1 upon 2 lambda. So, this as we know this is coming to be 2 upon lambda upon minus 1 upon 2 lambda.

Now, if we solve these 2 lambda we take this will become 4 minus 1 3 by 2 lambda that is 1.5 divided by lambda and we know 1 upon lambda is the MTTF of individual component. So, MTTF of system here turns out to be 1.5 times MTTF of individual component. So, if we have two components working together, MTTF is not doubled MTTF is around 1.5. And again, this all is applicable when we have the exponential distribution.

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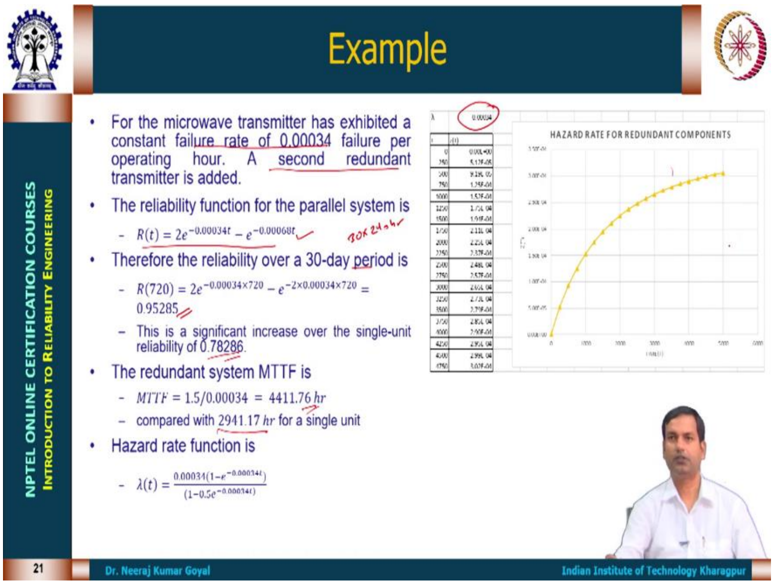

Now, let us say if you take one example, that microwave transmitter is there, which is having the failure rate of 0.00034, this we already considered, but now let us say we earlier we considered only one equipment. Now, let us say we add one more transmitter. So, there is a two-transmitter system where if one fails, another one will continue to do the function. So, in this case, what is the reliability of parallel system, reliability of parallel system is 2 e to the power minus lambda t minus e to the power minus 2 lambda t.

Then reliability for 30 days period we want to calculate, if you want to calculate for 30 days period, earlier also we have calculated so that will become 34 into 32 into 24 hours, that comes out to be 720 hours. So, for some because failure rate is given failure for operating hours, so, we have to convert the time into hour, we get 72 hours. So, e to the power minus lambda into 720 minus e to minus 2 lambda into 720. This gives me the relative 0.95285. Now, if you see this the earlier the reliability was for single unit was 0.78286, 78 percent around now it become almost fit 95 percent.

So, it is a very high increase in the reliability. And what is going to be the MTTF for new system, that is 1.5 of single system MTTF, single system MTTF or we can say 1.5 time divided by lambda. So, 1.5 divided by lambda gives us this. So, this is around 1,5 times of the single unit of the MTTF. Earlier it was around 2941. Now it is become 4411 and hazard rate function lambda t is given here, the same hazard rate function I have just taken from earlier and just by putting this hazard rate here, the same curve we are able to get.

And we see they are plotted this from 0 to around 47, 4800 around 5000. And we get this, if you see this is hazard rate, increasing and so in this increasing hazard rate is up to here. And further also it may increase a little bit, but it is asymptotically now becoming parallel to the x axis.

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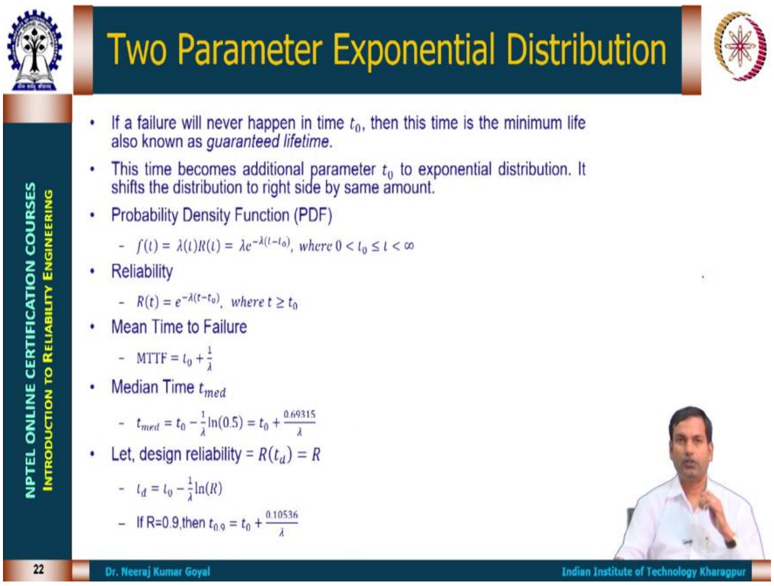

So, I will stop here. And we will continue our discussion in next lecture.