Introduction to Reliability Engineering
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Lecture 09
Two Parameter Exponential Distribution

Hello everyone, welcome to lecture number 9. So far, we have covered topics such as reliability definitions and basic probability concepts, which will be relevant in future lectures. We also explored the constant failure rate or exponential distribution. Today, we will introduce a variation of the exponential distribution called the two-parameter exponential distribution, which is commonly used in modelling the time until a certain event occurs. This distribution allows for a non-constant failure rate and has two parameters - the scale parameter and the location parameter. We will dive deeper into this topic in this lecture.

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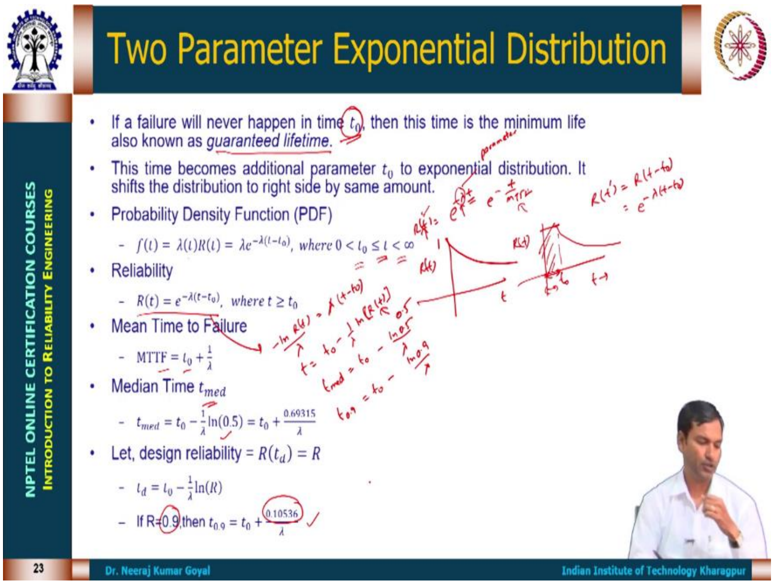

So, the two-parameter exponential distribution add additional parameter at time t0, what was the single parameter exponential distribution that was e to the power minus lambda t was the reliability equation for it.

$$f(t) = \lambda(t)R(t) = \lambda e^{-\lambda(t-t_0)}, \text{ where} 0 < t_0 \le t < \infty$$

So, here lambda was the parameter, whenever we talk about distribution there are two things one is the random variable for which the distribution is the function. So, the reliability when we say reliability against the value of the random variable. So, time to failure is a random variable. So, reliability is a function of time to failure. Similarly, hazard rate is also a function of time to failure or we can say the failure or probability unreliability is also a function of time to failure that is for this reflecting here.

$$R(t) = e^{-\lambda(t-t_0)},\ \text{where } t \geq t_0$$

Now, this time to failure will appear in this distribution in some form. So, that is t here. Now, but what are these parameters lambda etcetera. Now, what happens from system to system how the difference will be there, one system will have different reliability another system will have different reliability though they are following exponential distribution, but still they are following the different exponential distribution and that that difference is determined based on the value of the parameter.

So, one system having the one value of parameter lambda another system which is having another value of value of the parameter lambda both will follow the exponential distribution, but both will have the different exponential distribution because there is a different value of the parameter.

$$\text{MTTF} = t_0 + \frac{1}{\lambda}$$

So, once we determine the parameter for a particular component or the system that defines the system distribution. So, parameter is going to give the different instances of the same distribution. So, the earlier we have one parameter that is lambda. lambda is also called as the failure rate or constant failure rate. We can also write it as minus t upon MTTF where MTTF, though it is MTTF of the distribution, but again this is also parameter because it is dependent on the value of the lambda. So, now here let us say generally, our concern is that reliability at time t equal to 0 is always 1, and then it always keeps on decreasing by whatever amount applicable.

So, here we are discussing a specific case or some cases where we say that for time t equal to t0 that means, up to time t0 here, small time t0 here we have no chances of failure that means, up to this time or this is the minimum life, this is the life for which the component or the system is going to survive there is no chance 0 chance of failure means, it is not possible that the system will fail during this period, then this period we are calling this 0 to t0 we are calling as a additional parameter this additional parameter we can call it as the guaranteed lifetime, that means or we can say this is the minimum life. So, this is the time for which we have no failure no possibility of failure is there.

So, what will happen in this case then again reliability will start from one from here then again keeps on decreasing, same curve here what happens it gets shifted by the time t equal to t0. So, this shifting is happening because of this guaranteed life.

So, these additional parameters how do we take into account into the our equations, here reliability is represented as R same if we replace t as t minus t0 it becomes same. So, Rt dash that is a t dash is the new parameter then for this becomes t minus t0, that will e to the power minus lambda t minus t0. Similarly, if you differentiate this ft will be minus d Rt over dt or we can say lambda t into Rt. So, lambda t is nothing but lambda t is again lambda only.

So, lambda into but if you know this lambda is not applicable for 0 to t0 because there is no failure lambda t is 0 failure properties failure it is 0 no possibility of failure during this period. So, here the reliability function is defined only when t our time is greater than t0 and less than or up to time t equal to infinity.

So, here for t less than t0 it is kind of undefined or we can say it is equal to 1. So, here small ft comes out to be lambda into e to the power minus lambda t minus t0. Now, here similarly reliability is starting from t0 only reliability and first below time t0 we consider that there is no distribution this is a fixed value and MTTF. Now here if we have this reliability, when we integrate this from 0 to infinity dt we know that up to see same curve is there only up to here to here we have the additional so, this additional area which is added here.

Now, this is up to t0 this is 1 so, area under the curve that is small time t0. So, small time t 0 is added to the earlier MTTF earlier, MTTF was 1 upon lambda. Similarly, we can get the function because Rt is this. So, if we solve this for time t, then we know minus ln of Rt will be equal to lambda t minus t0, just taking minus ln on both sides and then if I divide by lambda will be gone from here then t will be equal to t0 minus 1 upon lambda ln of Rt.

$$t_{\text{med}} = t_0 - \frac{1}{\lambda}\ln(0.5) = t_0 + \frac{0.69315}{\lambda}$$

So, here if I want, if I am interested to know time t equal to t median. So, we know for t median reliability is equal to 0.5 at t equal to t median reliability becomes 0 to 0.5. So, t median will be equal to 0 t 0 minus ln of 0.5 divided by lambda we can get this using this and ln of 0.5 turns out to be around 0.7 that is 0.69315. Similarly, if I am interested to know what is my design life let us say I am interested to know design life means for a certain reliability.

$$t_d = t_0 - \frac{1}{\lambda}\ln(R)$$

$$t_{0.9} = t_0 + \frac{0.10536}{\lambda}$$

So, I am interested to know the design life for reliability to equal to 0.9. So, this can be obtained by placing 0.9 in terms of reliability that is t0. So, R t 0.9 design life will be minus ln of 0.9 divided by lambda ln 0.9 turns out to be minus 0.10536. So, that will become plus. So, similarly for whatever is our design life, we will be able to whatever if I my design life target is 0.9. So, I can know that what is what will be the time design life or design life which we can offer.

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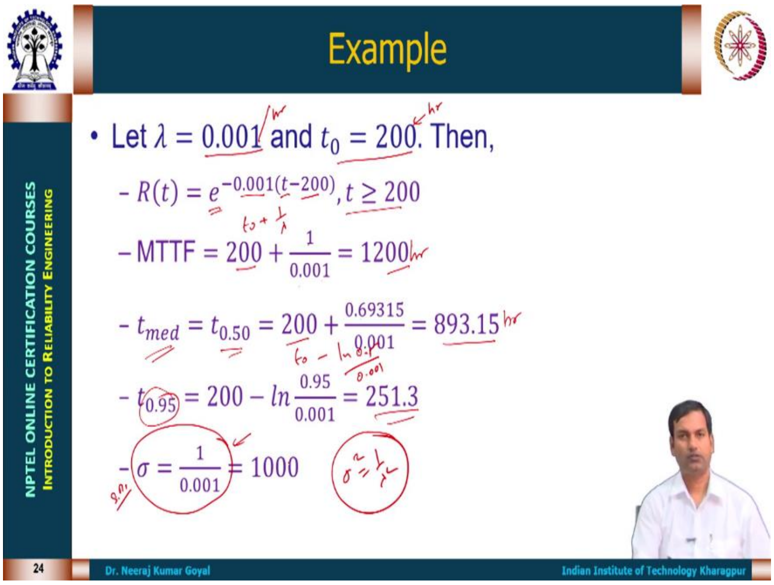
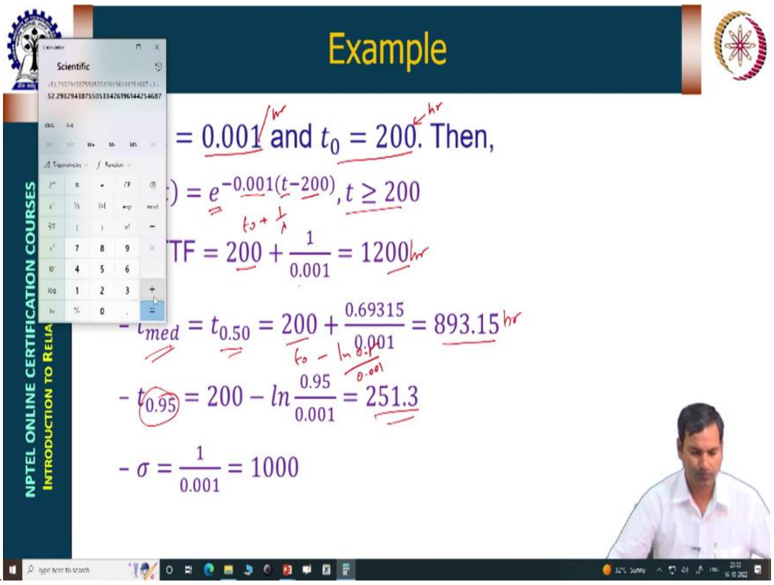
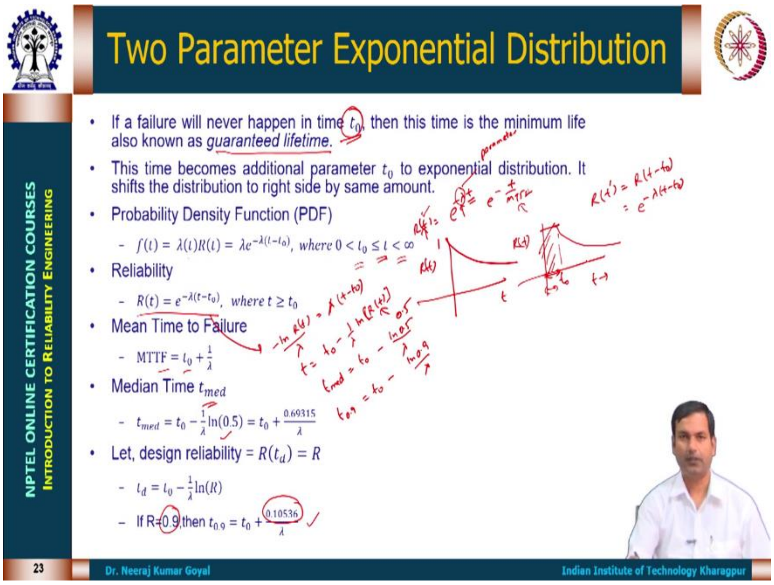


1 



Let us take one example that if let us say for the system failure rate is given us 0.001 the unit maybe per hour and again t0 is let us say 200 hours. So, we are interested to know what with the reliability function, so reliability function is exponential minus lambda into t minus t0 and t has to be greater than equal to this function is defined only when t is greater than equal to 200.

Same formula for MTTF and then we apply MTTF as we know that is t0 plus 1 upon lambda. So, 200 plus 1 upon 0.001. So, 200 plus 1000 gives the 1200 you can say 1200 hour, when we want to know t median as we discussed earlier t medium is t 0.5, reliability designs reliability 0.5.

So, this is t 0 minus ln of 0.5 divided by lambda 0.001 ln of 0.5 turns out to be minus 0.69315. This gives me the reliability of sorry t median of 893.15 hours, if I am interested to know the reliability that what is the design life for 95 percent reliability or R equal to 0.95. Same formula I have to use 200 minus ln or 0.95 divided by 0.001.

This if we solve we will get the 0.251.3, this like if we can calculate this using calculator if we use scientific then we have to calculate ln of 0.95 and we want to log CatLog of this that is 0.0519. If we multiply divide this by 0.001 and multiply by minus 1 sorry some mistake like 552.229 and once we add this to 200, that becomes around 252 points there may be some calculation based on calculation variations may be there little bit based on the calculators because, this is done by hand.

So, we many approximations may be carried out so, it comes around 252 and standard deviation sigma we can calculate, which is 1 upon lambda. So, as we see for two parameters also the sigma square is same sigma square is 1 upon lambda square, because variability same by changing the distribution to this to this the variability has not increased or decreased because this portion does not have any variability.

So, the variability essentially remains same and this is all applicable from time t equal to greater than t0. So, we comes out the variability comes out to be 1 upon lambda square or standard deviation as t turns out to be 1 upon lambda that is 1000.

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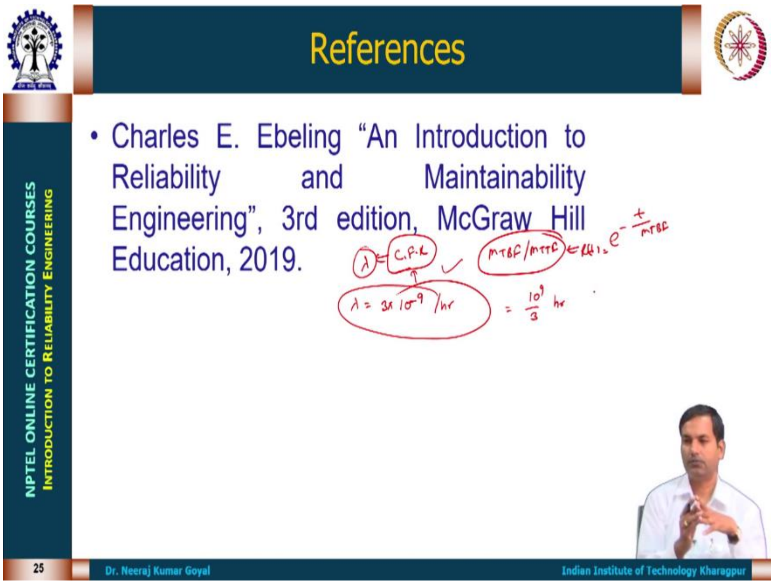

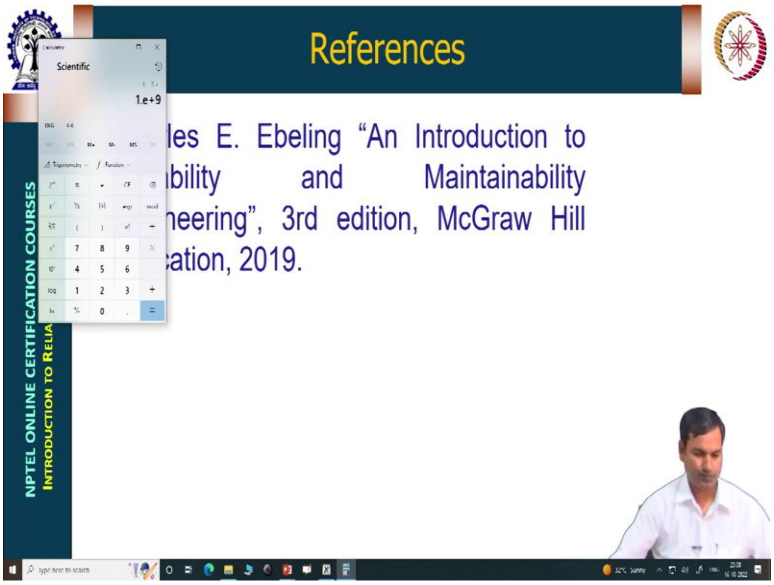
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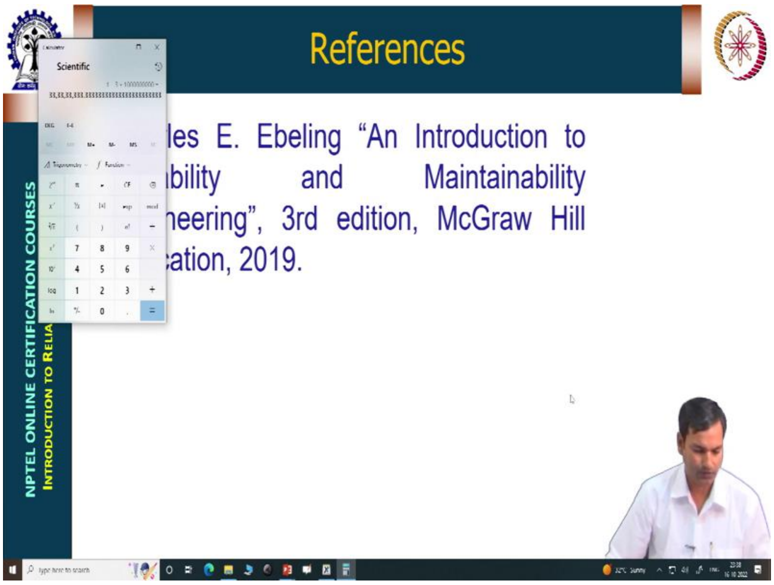
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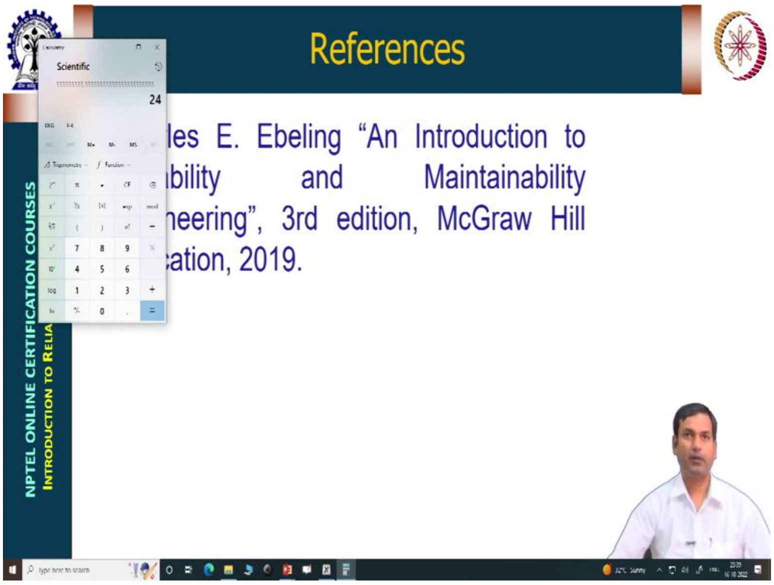
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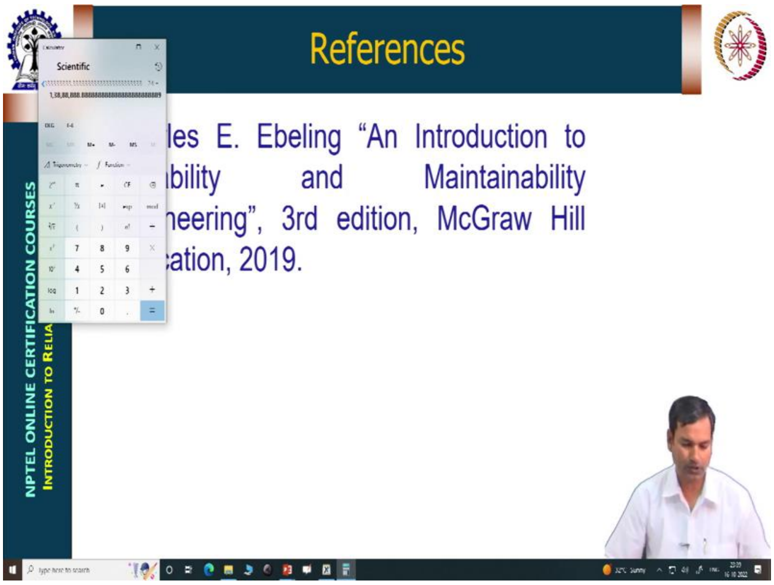
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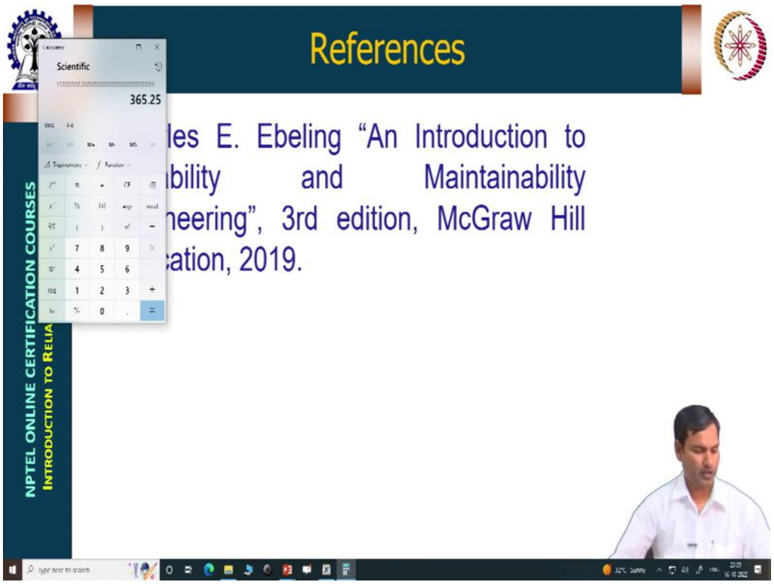
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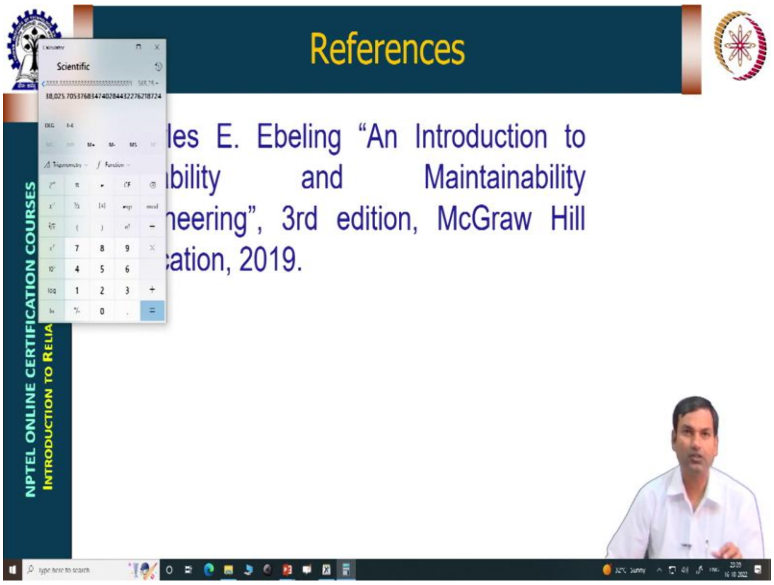
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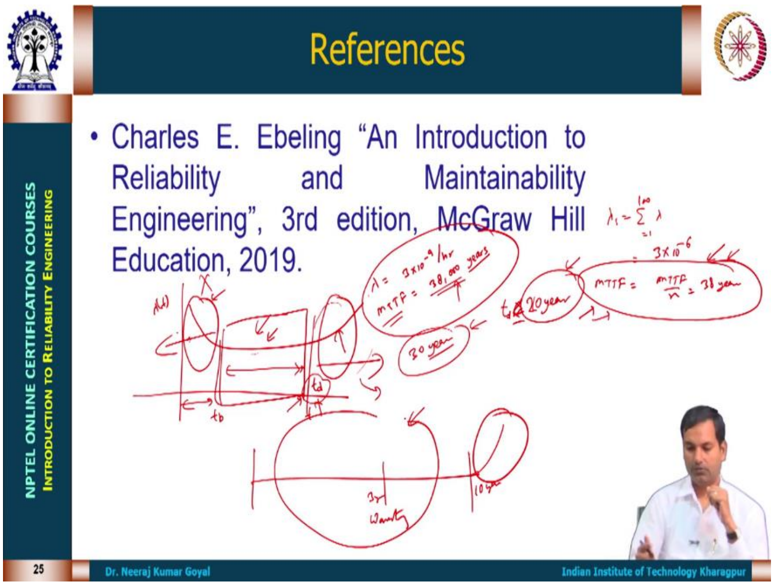

This all we have taken from this book and further discussion; we will continue for the time variant distributions. So, that will first discussion would take place from the verbal discussion, before going to verbal discussion, let us have one important discussion on this lambda.

So, what is this constant failure rate and if you see in the professional world there is a lot of disputes, though as I as we discussed earlier all the standards processes and even though if you refer to any product manufacturing catalogue, they would say one quantity that is MTBF or MTTF. They will tell that their component or part is having a certain MTTF.

So, by default, we are supposed to assume that this is following the exponential distribution. And if we want to calculate reliability, it will be for a given time t if I want to calculate reliability this will be e to the power t upon MTBF. Now, the question arises here that by default we are assuming that the system is not going to deteriorate.

So, or let us say, if we see the component handbooks and if we see the component MTTF etcetera, they will say that failure rate for the component is given as 10 to the power 3 into 10 to like for capacitors etcetera or registers you will see that failure rate is somewhere around 3 into 10 to the power minus 9 per hour.

So, failure rate per hour is this much is okay. But if I convert this into MTTF this will turn out to be 10 to the power 9 divided by 3 hours, which is a big number. If we see the number in another sense let us see this from the Excel. So, as we were discussing that we were interested in 1 divided by 3 into 1 e 9. So, if you see that number is quite large we can see in so, many three’s are there, 1234 at around 33. Now, let us say we divide this by 24. If you divide by 24 we will get the number of days, number of days that is around 1,00,000 around 1,38,00,000.

Now, if I divide this by 365.25 I will get years. So, if we see the years are coming out to be somewhere around 38,000. So, if you see when I have taken this MTTF my lambda was 3 into 10 to the power minus 9 per hour. In that case, my MTTF turns out to be somewhere around 38,000 years. So, it raises a natural question that what does this MTTF means is the cam component going to survive 38,000 years, the answer is definitely no the company know component can work for this long period of the time.

So, what does it mean in that case? Generally, to understand this, because there are many people you will find there they are saying that this because of this this MTTR becomes useless. But generally in practical sense, why it is still useful and because of the simplicity it again makes a lot of these possibilities which would have not exist if we were not taking this as a constant failure rate. So, but is it still valid.

So, the validity question comes into picture again and again, we know that generally we may have time and we may have the failure rate, which is following the this kind of bathtub curve. Now, this bathtub curve we have this constant failure rate.

So, let us say this is my life at which deterioration start. So, let us say this is my design life. And this is my life which is at which we I am doing the burning that is a tb. So, what happens generally though, our concern is that failure rate is changing here failure is changing here. So, what is limiting the life, most of the time life is being limited by the this phenomenon, whenever increasing failure rate starts what will happen majority of the components will start failing or here is the place where the failures will start.

So, but what happens generally this life, this life maybe 30 years only for a component like this, while I am writing 38,000 here, the life may actually be around 30 years, that means around in 30 years the deterioration will start and my component failure will start failing very quickly that means in around 30 to 50 years most of the most of the components may fail, that is because we are talking about this region.

But what happens in most of the time we are not going to use the equipment beyond up to 30 years that equipment may be used only for 10 years. The equipment in which I am using this component that equipment is used only for 10 years. So, for 10 years, my validity of the reason is only this, that is the constant failure rate that does not go to the, this region that is my increasing failure rate.

And this failure rate which is initially decreasing failure rate which can also be the responsible for that is already by choosing better manufacturing methods and by choosing the burning methods segregation and doing the same stressing and then removing the failed equipment. Once we do that, what will happen during this initially years also this is eliminated. So, this infant mortality period or burning period is also those failures are not also sent to the people.

So, people when they are using a component, they see only this constant failure rate region. But for constant failure rate, how much will be the component MTTF or how much will be the lambda that will be this one that we have calculated. The condition here is that the component is not used beyond time td, if we consider that component is used beyond time td, then my failure distribution which was here that failure distribution which was constant will not be valid in that case my calculations will also not be valid.

So, this calculation is valid then design life is up to 10 years or 20 years or 30 years. But if it goes beyond that 30 years or beyond or whenever the we see that for different devices it may be different when the deterioration is start.

So, before deterioration takes place. The constant failure rate assumption is very much valid and this high MTTF and lambda is also valid because this lambda as we know for system, if I am calculating lambda as now, let us see I am using almost 1000 components like this. So, when I let us say if I include i equal to 1 to 1000 lambda.

So, what will happen in that case the lambda will be 1000 times. So, lambda will become system lambda will become 10 to the power minus 6 and MTTF will become 1000 times less that is MTTF divided by n. So, this will become 38 years. If I say some components if let us say not all components are falling in 10 to the power minus 9 some components will be 10 to the power minus 8, some will be 10 to the power minus 7 also.

So, in that case if let us it is a 10 to the power minus 8 component this will be around 3.8 years. So, system MTTF which is of our concern because the system will fail the system failure probability is going to be high, system MTTF is only 38 years here. So, that means, system we expect to fail before much within this time that 38 years we are expecting around 63 percent failure.

So, here when we see this that failures are happening before this period itself. So, this becomes that the calculations which you have done becomes valid here, because for the system level the total time or MTTF comes out to be quite low, it may be some 4 years, 5 years because multiple components are there, multiple failure rates are there.

When we sum that up, then this MTTF comes out to be in which is much less than this period design life. So that makes the sense because then it provides us the comparison purpose, because within this period, we expect that this component will fail with this failure rate. So here like when people are asking validity, whether we should use this constant failure rate or MTTF or no, I would suggest that we should use but we should use, but we should aware that these calculations that MTTF of 38,000 years does not mean that this is going to survive for 38,000.

Here it means that MTTF a 38,000 will be used when component is used only for 20 years or 30 years of before the deterioration takes place. So, before deterioration sets before that time, if we consider the failure rate of if we consider the MTTF that, MTTF is valid. But that is not indicative of life in that case for the component, but since most of the time the system lives or system MTTF would be lesser than the time before which that come this component sets in or this component is start deteriorating. The constant failure rate or this kind of high MTTF can still be valid for the component level.

Because at the system level then we will have the meaningful result at system level 38 years or 3.8 years, if it was 8 minus 8 it would have been 3.8, we have some valid outcomes, this is quite useful, because this provides us because most of the time in failure in industry when you see as a manufacturer or as an employer employee, in any company, which is producing any goods, consumer goods or certain goods, you will see that you have certain life for which you are liable like warranty life, then certain life you perceive that the system is supposed to work because after there is some critical or costly component starts failing and system becomes an unsupportive or the technology itself is supposed to improve.

So, let us say the warranty life you are giving around 3 years and you may have component system like around 10 years, let us say we talk about a free refrigerator or if we talk about our TV, sorry, not TV, TVs are much less life.

So, in this time, what will happen during this period, we will be able to have some meaningful and the cost the kinds of failures the system will see will not be life related failures, those will be the random failures belonging to this constant failure rate reason, because of random failures, this assumption of having exponential distribution, because this is the area of concern. Once the failure starts let us after 10 years, when multiple failures starts happening, we are not going we are not having an intent to use the system in this region, we are going to use the system in this reason where the failure rate is constant.

So, when we do the design and especially whenever these failures, these life related failures are generally perceivable you can predict them and you can find out a safe life. So generally, what happens there is a preventive maintenance policy. So, before these components actually starts deteriorating or preventive maintenance policy sets in and before deterioration actually can lead to the failure these components are discarded before the failure itself.

So, because of that population, because we are taking out this population also, we are taking out this this population we are addressing by the by name, this population we are addressing with the preventive maintenance.

So, effectively most of the time, we are working with the components or the parts which are setting in this, and our concern is mostly this random failures. So, here these models provide us a very good assessment to take these random failures into account and understand that though, we are not able to foresee any failure, but still there is a probability of failure lying with the systems. So, I will stop it here today. This lecture will continue in next lecture we will continue with the time dependent failure distributions. Thank you.