Thermodynamics
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Lecture 48
Tutorial Problems – Part 1

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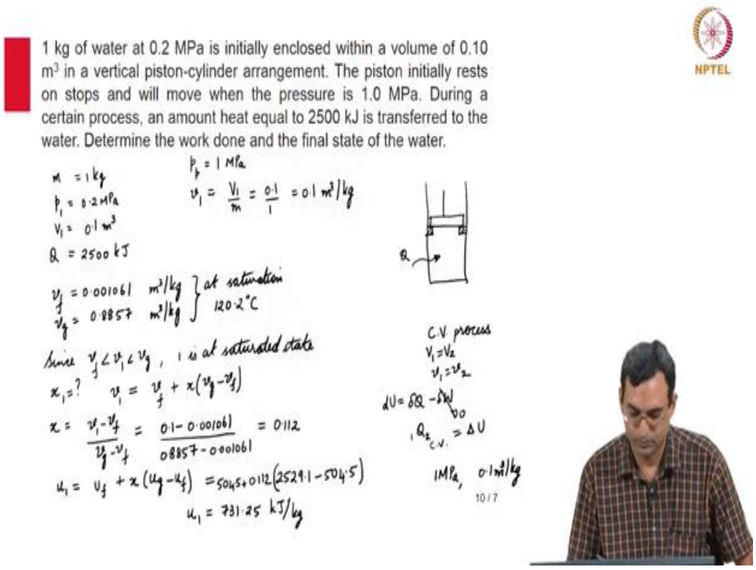

Figure 1.

Solution of the problem in Fig. 1:

$$m = 1\,kg, V_1 = 0.1\,m^3, p_1 = 0.2\,MPa, v_1 = \frac{V_1}{m} = 0.1\frac{m^3}{kg}, Q = 2500\,kJ$$

The piston moves when the pressure becomes 1 MPa. Hence, the pressure outside plus the pressure due to the weight of the piston equal 1 MPa.

The initial state is $p_1 = 0.2\ MPa, v_1 = 0.1\frac{m^3}{kg}$

At 0.2 MPa, $v_f = 0.001061\frac{m^3}{kg}, v_g = 0.8857\ \frac{m^3}{kg}$ (from steam tables). Hence, $v_f < v_1 < v_g$. Thus, at state 1, we have a mixture of water and vapour (saturated mixture).

Hence, the quality $x = \frac{v_1 - v_f}{v_g - v_f} = 0.112$

Until the pressure becomes 1MPa, the process is a constant volume process. Let's calculate the heat required for this constant volume process between $p_1 = 0.2\ MPa, v_1 = 0.1\frac{m^3}{kg}$ and $p_2 = 1\ MPa, v_2 = 0.1\frac{m^3}{kg}$.

The first law for this constant volume process in the integrated form, $\Delta U = Q$ (W = 0 and there are no changes in kinetic and potential energy)

Now, $Q = \ \Delta U = m\Delta u = m(u_2 - u_1)$ ………(1)

Now, $u_1 = \left[u_f + x\left(u_g - u_f\right)\right]_{0.2\ MPa} = 504.5 + 0.112(2529.1 - 504.5) = 731.25\frac{kJ}{kg}$

At $p_2 = 1MPa, v_f = 0.001127\frac{m^3}{kg}, v_g = 0.1944\frac{m^3}{kg}$. Hence, $v_f < v_2 < v_g$. Thus, state 2 ($p_2 = 1\ MPa, v_2 = 0.1\frac{m^3}{kg}$) lies inside the liquid-vapour dome.

Now, $x_2 = \frac{v_2 - v_f}{v_g - v_f} = 0.512$ (The quality has increased in this process. It means there is more vapour now compared to that at the initial state.)

So, $u_2 = \left[u_f + x_2\left(u_g - u_f\right)\right]_{1MPa} = 1693.9\frac{kJ}{kg}$.

Now, (1) implies $Q = 1(1693.9 - 731.25) = 962.65\ kJ$.

Out of the total heat supplied, i.e., 2500 kJ, 962.65 kJ is used up in a constant volume process raising its pressure from 0.2 MPa to 1 MPa (temperature also rises). The remaining heat, which is 2500-962.65=1537.35 kJ, is going to be used in a constant pressure process.

Writing the first law, $dU = \delta Q - \delta W = \delta Q - pdV$.

For a constant pressure process, $dU + pdV + Vdp = \delta Q \rightarrow dH = \delta Q$

Integrating, $Q_{2-3} = \Delta H = m\Delta h = m(h_3 - h_2)$.

Now, h_2 is the enthalpy at the state where p=1 MPa and v=0.1 m^3 (x_2=0.512).

Hence,

$h_3 = \frac{Q_{2-3}}{m} + h_2 = \frac{1537.35}{1} + h_f + x_2\left(h_g - h_f\right) = 1537.35 + 762.5 + 0.512(2014.6) = 3331.32\,\frac{kJ}{kg}$.

The state 3 is at 1 MPa and h_3 = 3331.32 kJ/kg. At 1 MPa, h_g = 2777 kJ/kg. Since, $h_3 > h_g$(at 1 MPa), the state 3 is in the superheated region.

We are also asked to calculate the work done. The work is done only in the second phase of the entire process where pressure was constant.

Writing again the first law for the constant pressure process in the integrated form,

$\Delta U = Q_{2-3} - W_{2-3} \rightarrow W_{2-3} = Q_{2-3} - m(u_3 - u_2)$ …..(2)

We need to calculate u_3. h = 3331.32 kJ/kg lies between the enthalpy values at 400 °C and 450 °C in the table for superheated steam at 1 MPa. Hence, interpolation gives,

$$\frac{T_3 - 400}{450 - 400} = \frac{h_3 - h\ at\ 400\ ℃}{h\ at\ 450\ ℃ - h\ at\ 400\ ℃} = \frac{u_3 - u\ at\ 400\ ℃}{u\ at\ 450\ ℃ - u\ at\ 400\ ℃}$$

We know h_3. Solving the above equation, T_3 = 431.3 °C and u_3=3009.85 kJ/kg.

Hence,(2) implies $W_{2-3} = 1537.35 - 1(3009.85 - 1693.9) = 221.4\ kJ$.

The entire process on a p-v diagram is shown in Fig. 3. State 3 is the intersection of the isotherm T_3=431.3 °C and the isobar p=1 MPa in the superheated zone.

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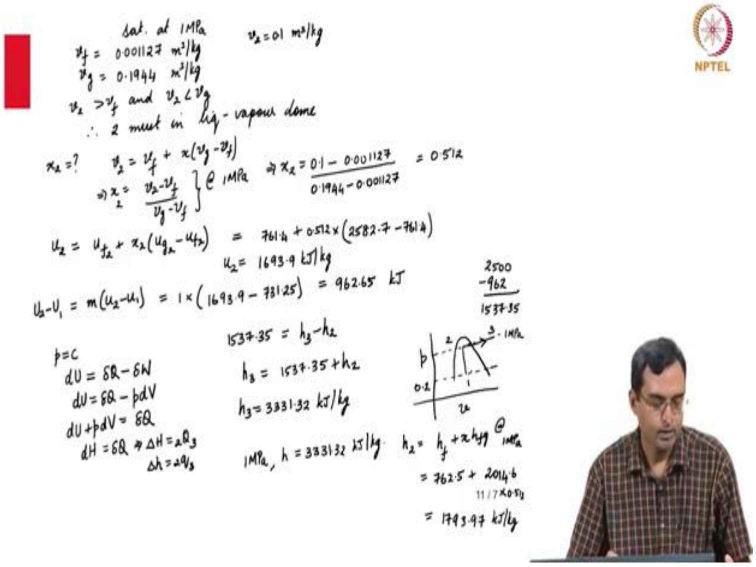

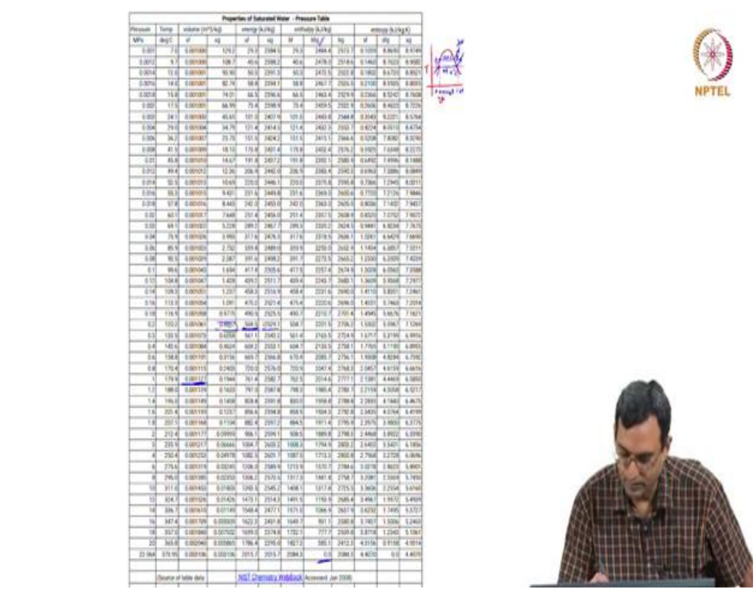

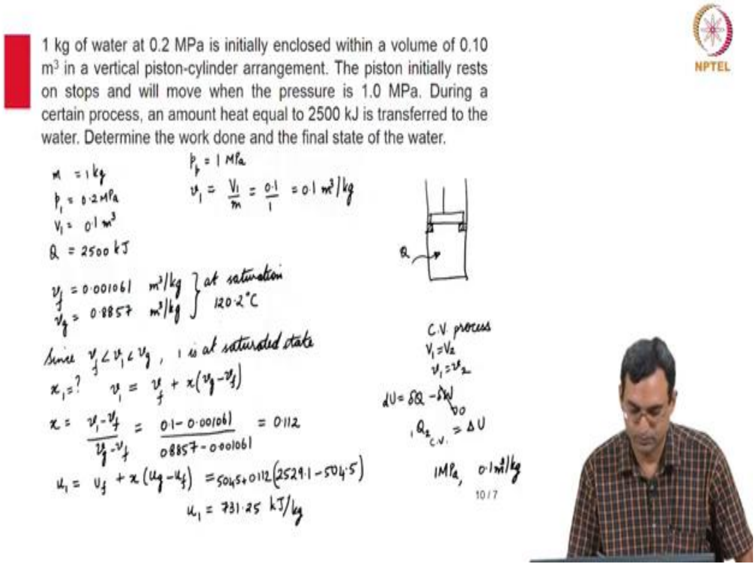

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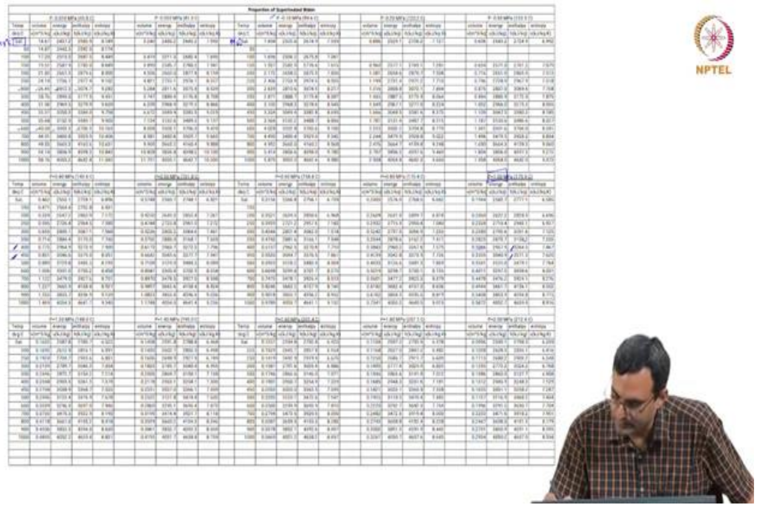

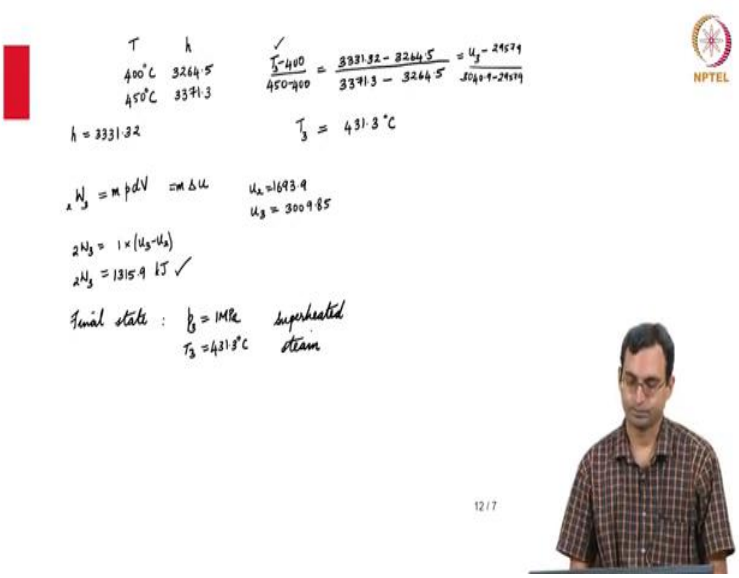

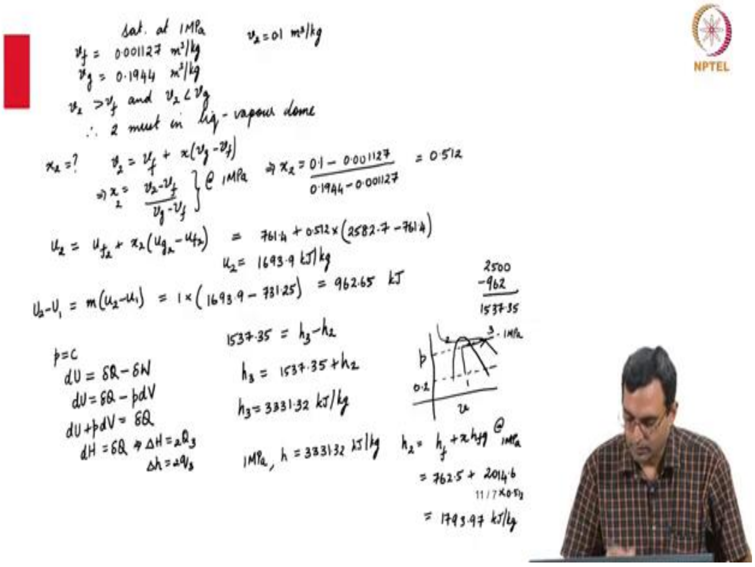

Figure 2.