**Micro Foundations of Macroeconomics**

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**Lecture – 07**

**Two Period Model II**

Welcome back. So, we started understanding the 2-period model and in the 2-period model, we derived the lifetime budget constraint in the last session. We will have a similar reference here.

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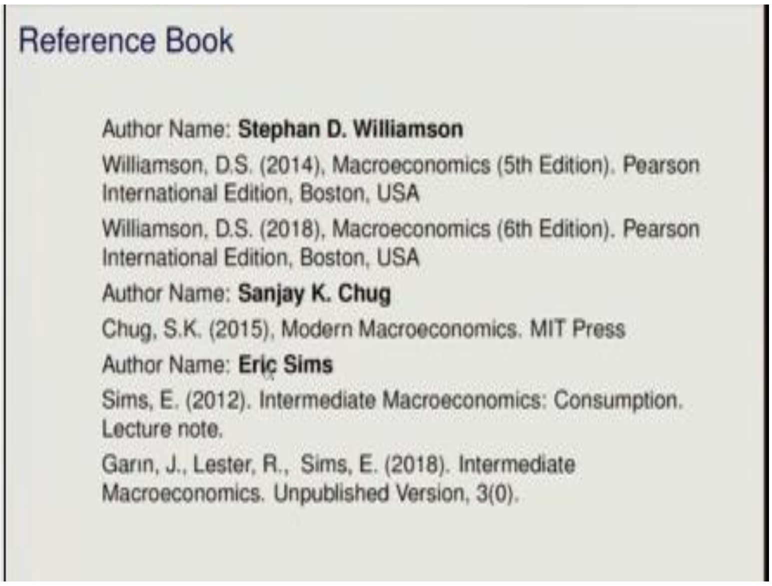


It will remain the same not much change, we will be having the same references Eric Sims is going to be the major reference, and Williamson. So, you can refer these 2 books apart from Sanjay K Chug that we have.

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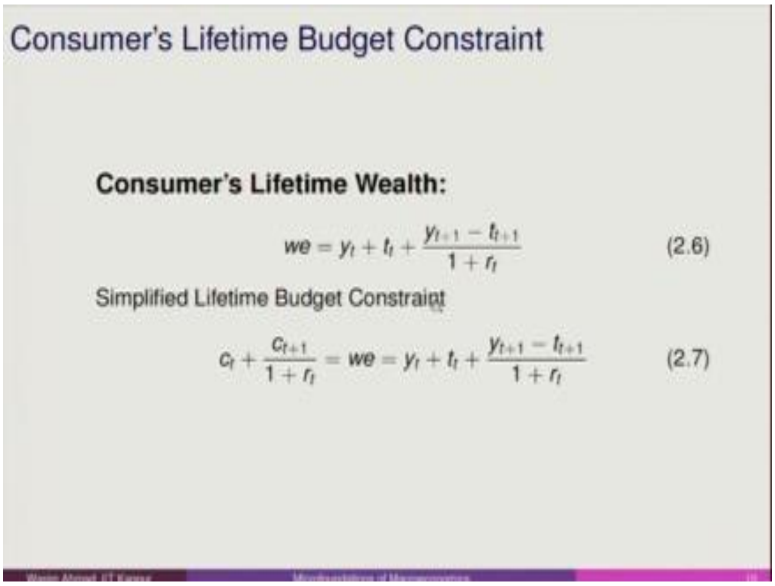


So, we have derived this part in simple steps, we derived the first period then we derived the current period then we derived the future period, so, one period and second period. We derive the saving because in the first period he is saving some amount and then we superimpose this into the budget constraint of this. So, here, if we can simply go for substituting this here in the current period. Then you can have,

$$c_t + \frac{c_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t} = y_t - t_t$$

And finally, we are able to derive this part which is the lifetime budget constraint. So, this is how it looks like

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1+r_t}$$

Now, we have to understand certain dynamics, what are those dynamics? One is that when we are having such a type of scenarios which is having the future period income (Refer slide 11 figure). Here you have the future consumption. On this side you have the current consumption, and this is the budget line of the representative consumer. So, an endowment in the current period is this $we$ and in endowment in the future periods incoming endowment wealth and endowment in the future period is we multiplied by $1 + r$. Here instead of t + 1, we are just focusing on transpose.

Here you have to know 2 things. This particular bracket from this to this B to E. The representative consumer is a lender and from E to A the representative consumer is a borrower. When do we say that? So, here we are defining the boundary of this representative consumer that this representative consumer is having $y' - t'$ and then here it is $y - t$. Now, if we go for suppose, if I am saying that this representative consumer is here.

So, consumption is this much which means that the future consumption is this much and the current consumption this much which means that this representative consumer will have to go for extra amount of borrowing. This means that if I am drawing a line here, like this then this is leading to what is called the borrower. A borrower in the sense that his future period consumptionis less than the endowment.

So, in the current period here it is the lender and once it is lender then why it is lender? Because his current period consumption is less than the endowment.

Which means that this much he is saving and this he can transform into the future period consumption. So, the amount that goes to the future period suppose, this much. So, this is the saving that he has and he can go for lending it. This amount can go for and then it will become $(1 + r_t)s_t$. Let me give you one more example. And this is important to understand.

When I am saying that the consumer is a lender then here it is borrower. This is the income threshold that we are defining for this representative agent. The moment I say that it goes beyond this, it comes this side then here it becomes the extra borrowing for him because this is the boundary line for the consumption so, moment he moves this side, it is a borrowing zone, the moment he moves this side, it is the lending zone.

How we are moving the side? If we suppose this representative consumer is consuming this much, in the future period and in current period he is consuming this much which means this much amount of current income is saved which can be transferred through the future period. So, this becomes really important to understand. And second thing, if we want to take in calculate the slope of this.

So, you can simply go for $c_{t+1}$upon $c_t$ it becomes – $(1 + r)$ it is easier. So, slope of this is – $1 + r$. This is what we tried to defineSo, I think I hope the lender and borrower case makes easier to understand.

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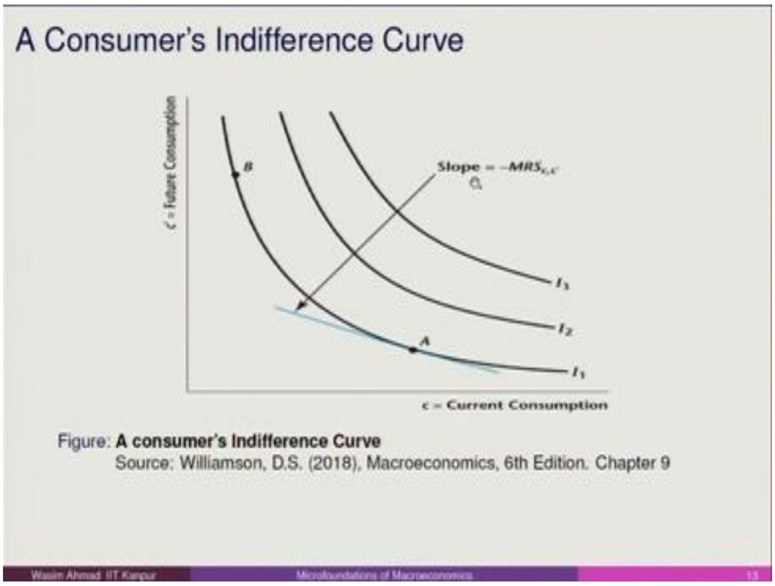

Then here we have the indifference curves. So, I think it remains same that we have done in the one period model that there also we have assumed the same thing that here you have the future consumption and here you have the current consumption. It is having the well–defined convex shape which means that more is better. The only thing that you have to keep in mind the properties of the indifference curve.

That 2 indifference curves will not intersect. On the indifference curve wherever you are, the utility remains same doesn't change. So, those conditions will satisfy and the budget constraint variable it will tangent to the indifference curve that will be the point at which he or she is having the optimal level of utility.

So, in terms of current and future consumption, those things are same here, not much change in preference also, the only thing that we have to make sure that he is we have to arrive at the point where he is indifferent about choosing current and future period consumption. So that is the, trick that we will be using it here.

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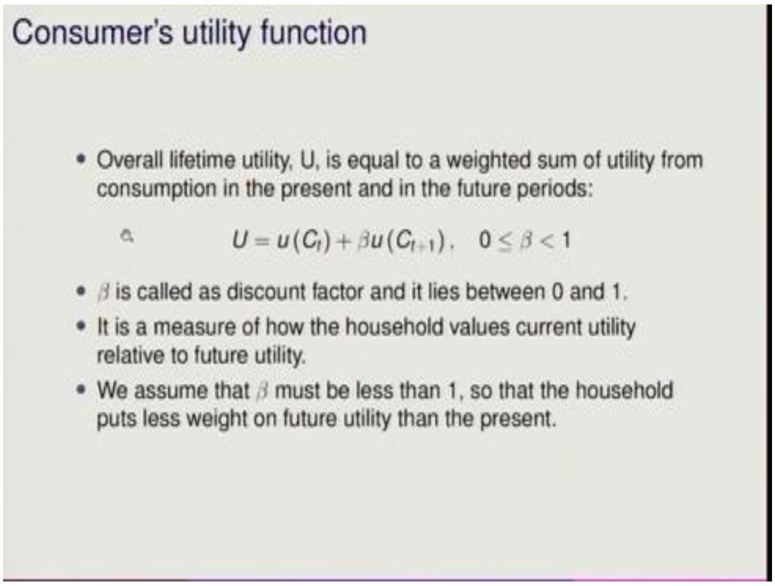


Now, here we will be working on the utility function. Unlike the one period model, where we had C l here, we are having 2-period model, so here we have,

$$U = u(C_t) + \beta u(C_{t+1}), \qquad 0 \leq \beta < 1$$

So, we have to maximize this with respect to $c_t$ and $c_{t+1}$.

And $\beta$ is what we have the behavioural coefficient which means that how much this particular agent is going to give more weightage to the future than the current. If the beta is higher, more weightage to the future, if beta is lower, less weightage to the future. So, this beta the unknown parameter is going to decide about the preference of sorry future to current period.

So, it is also the measure you can say in I would say in a different way also. So, here it is called discount factor it lies between 0 to 1. It measures how the household values current utility related to future utility which means that how this particular representative agent is going to decide about whether he has to save more in the current period for the future or he should be utilizing all his income in the current period itself.

We assume that $\beta$ must be less than one because if it is one then there is no point in investigating the so, equal to examine. So, this is how we try to put this weight.

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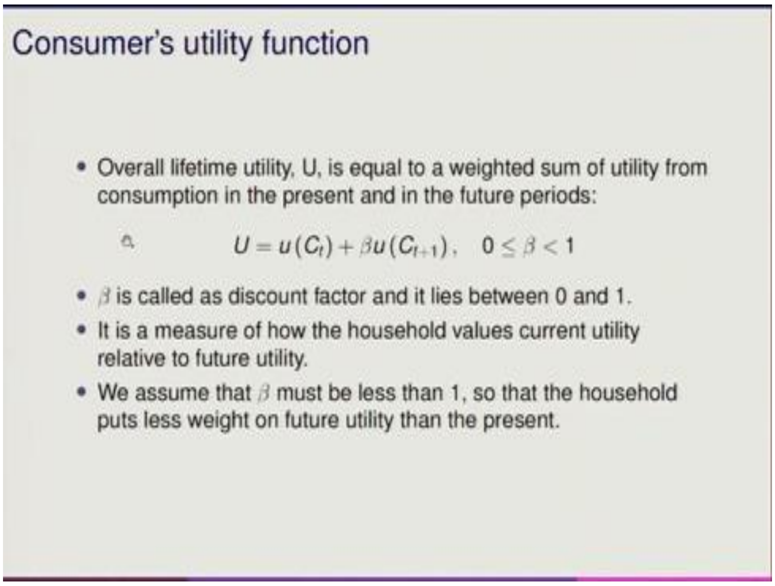


Now, we have already gone for lifetime budget constraints. In lifetime budget constraint, here we have

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1+r_t}$$

So, now, we can go for the optimization condition. In case of 2 period model the optimization condition has a lot of meaning.

In what situations this representative consumer can be indifferent about the current and future period consumption. So, for that we have,

$$\max_{c_t, c_{t+1}} U = u(C_t) + \beta u(C_{t+1})$$

Subject to

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

So, here you can go by using both the methods. Either you can go by Lagrange multiplier.

Or you go by method of substitution which means that you solve for $c_{t+1}$and then substitute it in $c_t$ with respect to all in $c_t$ terms and then you go for the direct differentiation or you can go apply for Lagrange multiplier and go for partial differentiation. So, it is all possible here. So, let us work it out. So, here I go by the method of substitution. So, here, what we do is that we solve for $c_{t+1}$ here from this particular equation and then we try to substitute.

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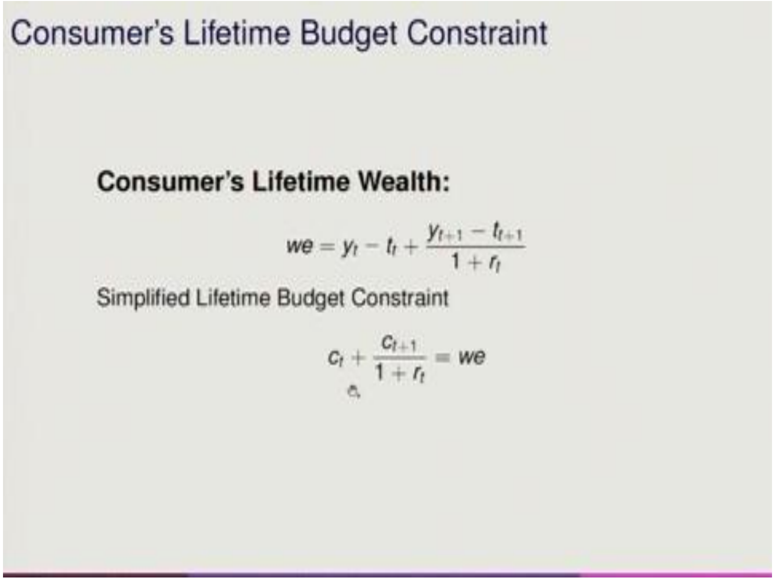


So, if you solve for $c_{t+1}$this is what we are going to get. So, here $c_{t+1}$ it becomes,

$$C_{t+1} = (1 + r_t)(Y_t - C_t) + Y_{t+1}$$

I substitute the $c_{t+1}$in this and this is how we get

$$\max_{c_t} U = u(C_t) + \beta u((1 + r_t)(Y_t - C_t) + Y_{t+1})$$

and to find the optimum I go for differentiation. So, the first order condition of this will be this.

$$\frac{dU}{dC_t} = u'^{(C_t)} - \beta u'((1 + r_t)(Y_t - C_t) + Y_{t+1})(1 + r_t) = 0$$

Since,

$$C_{t+1} = (1 + r_t)(Y_t - C_t) + Y_{t+1}$$

So, this is the expression that we get for $c_{t+1}$, if we are going to solve from here.

$$u'(C_t) = \beta u'(C_{t+1})(1 + r_t)$$

So, these I write it here. So, here we have what is the condition that we are getting? Here we are getting the condition of marginal utility of current period consumption is equal to beta times the marginal utility of future consumption. And it is also multiplied by the rate of return that he is going to get when he is saving some amount of income in the current period which he is going to get in the future period.

It is coming from the representative agent that if you remember here, I had shown you that he is going to get some amount of income in the future period.

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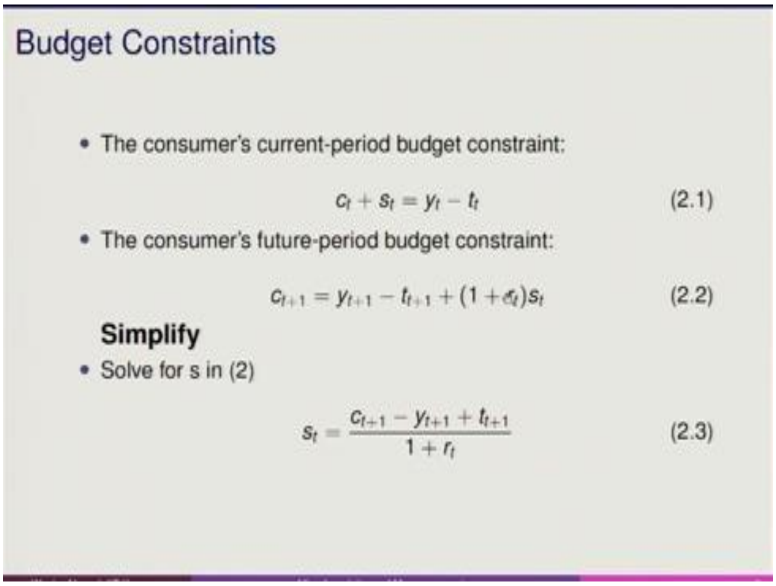


So, this is the $1 + r_t$, here it is coming $1 + r_t$ because whatever amount of money that he is going to save in the current period, he is going to get $1 + r_t$ in future. This is what we have $1 + r_t$. Here we have the marginal utility of current period consumption is equal to beta times multiply by one beta times $1 + r_t$ times the marginal utility of future consumption.

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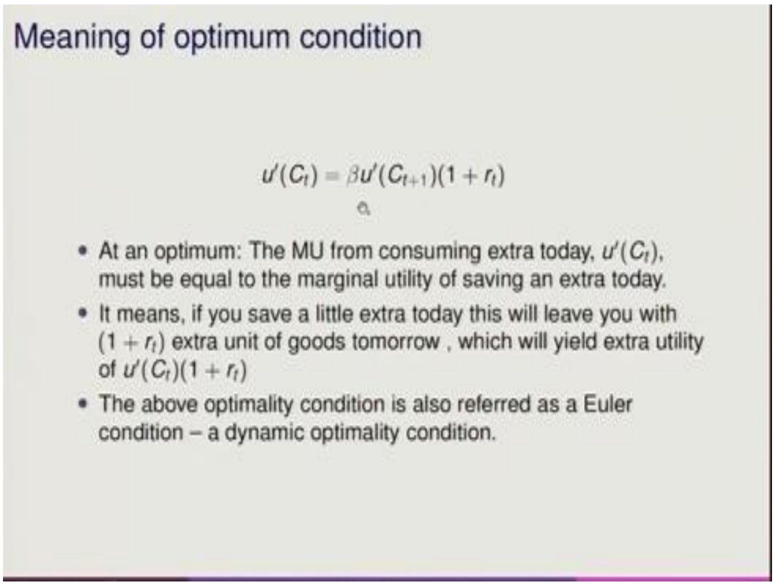


Now, what is this meaning? So, this particular condition has a lot of applications in macroeconomics and lot of time when you try and understand the when you work with the very advanced macroeconomic models. For example, the dynamic general equilibrium models and then you try to understand from the stochastic perspective. So, if you try to induce the stochastic model.

Then it began dynamic stochastic general equilibrium model. In macroeconomics it is very common. So, DSG models are quite popular these days, these models help you understand with the similar kind of setup and try to get the inference about the macroeconomic picture by introducing different agents. So, this particular course is helpful to understand that kind of dimension of DSG.

So, it will give you the brief idea about very, I would say very mature idea about the DSG model and then you can further use such formulations in your DSG modeling formulation. So, unlike macroeconomic models, where you have a lot of applications of simultaneous equation models, where we try to work with the large set of simultaneous equation we identify and then we try to estimate the model.

People have gone for understanding the dynamic stochastic general equilibrium model where they theoretically build the different agents and try to define in theoretical context and then they try to calibrate that with certain formulations. So that kind of understanding has become a quite a mainstream economic macroeconomics these days. Such type of micro foundations is important to understand these dimensions.

Otherwise, IS and LM framework or even the basic macroeconomics, though it gives you the feel about the functioning of the macroeconomic indicators but in the context of the recent developments, if you try to see then you will not have idea that where are we heading. So, these days a lot of theoretical aspects about the macroeconomics are being examined and in that context, this particular textbook, this particular course, is going to be helpful.

So, here the marginal utility of current consumption is called $\beta$. Here you have the, marginal utility of future consumption multiplied by this, we call it that if it means that the marginal utility of current consumption is equal to marginal utility of future consumption which means that future consumption multiplied by the, rate of interest that you have. Which means that your future consumption, if you are how much you are saving.

Or whatever you are consuming today. It means whatever you are consuming today, it is equivalent to saving that extra unit in future. at this level the consumer should be indifferent that how much he has to say subject to this behavioral coefficient or discounting factor this we have. It means that if you save a little extra today, this is the marginal utility.

This will leave you with $1 + r_t$ extra unit of goods tomorrow because this is the reward that you have for the future consumption which is going to yield you extra utility which is having this part. If you are having 100 rupees, if you are going to save 40 rupees, today, you are going to get 40 into 1 + r amount in a future. So, if you have planned to save some amount, so, this amount will be transferred to future.

So, it is up to you either you consume today or the future. If you are consuming it today, it will be simply the marginal utility of current consumption. But if you are saving that amount, it is equivalent. So, this particular part, it becomes really important to understand that marginal utility of current consumption it is equivalent to the marginal utility of future consumption multiplied by the behavioral coefficient beta and the rate of interest earnings that you are going to have. In the dynamic optimization context, this particular equation it is called the Euler condition. So, Euler condition it is having a lot of meaning in macroeconomics and even in the field of dynamic optimization. And here, this equality has a lot of meaning. So, this equality can be interpreted in a different way also. So, finally, marginal utility from consuming extra today must be equal to the marginal utility of saving an extra today which means that we are saving then you are going to get this much amount in future.

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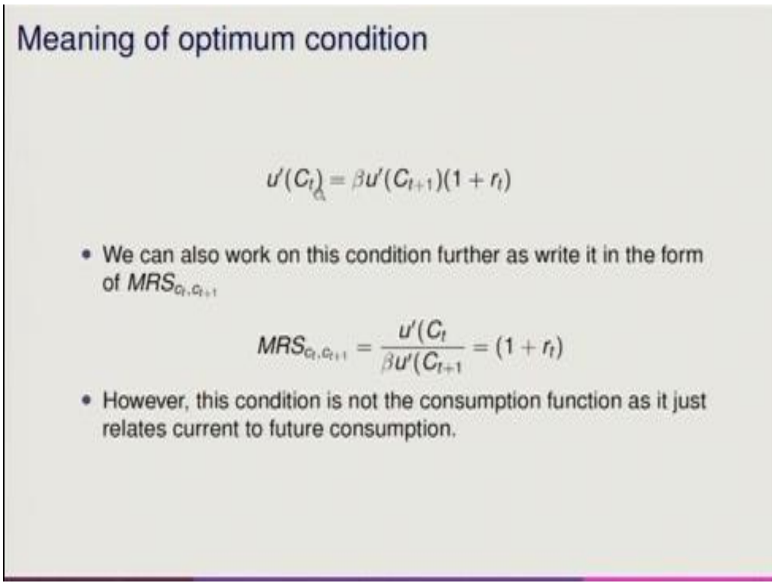


So, if I am going to drive the marginal rate of substitution which is the ratio of marginal utility of current to future, so, how does it look like?

$$MRS_{C_t,C_{t+1}} = \frac{u'(C_t)}{\beta u'(C_{t+1})} = (1 + r_t)$$

This means that the ratio of current to future consumption it is equivalent to the future interest earnings that you are have. So, reward that you are going to get. But, keep in mind that we are talking about consumption. But consumption I said that we study in macroeconomics, this is not this part, this is the micro foundation.

So from here, we cannot say that this is the consumption function because this is not the consumption function. Consumption function is equivalent to when you have the consumption dependent upon current and future income and rate of interest. So that part will be taking up later. But here, this particular condition helps us understand the 2-period scenario in a better way that given the current and future income.

How much this representative agent will have the opportunity to either consume one unit extra unit today, or it should be equal into what he saves that unit today. So that he gets extra or $(1 + r_t)$

on that unit in the future. So you have to think in that dimension in terms of difference. Not in terms of the consumption function that we normally assume in case of consumption theories.

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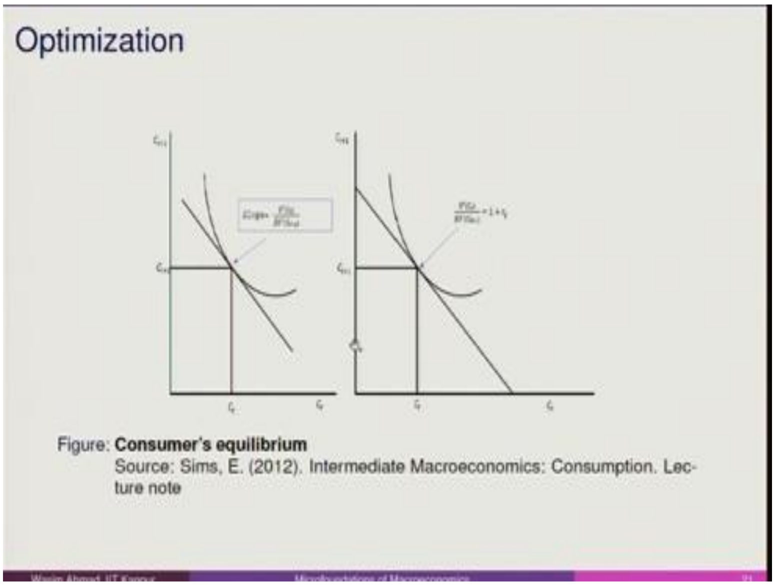


So, if you think about this slope, this is how it looks like. So here we have the current and here you have the future.

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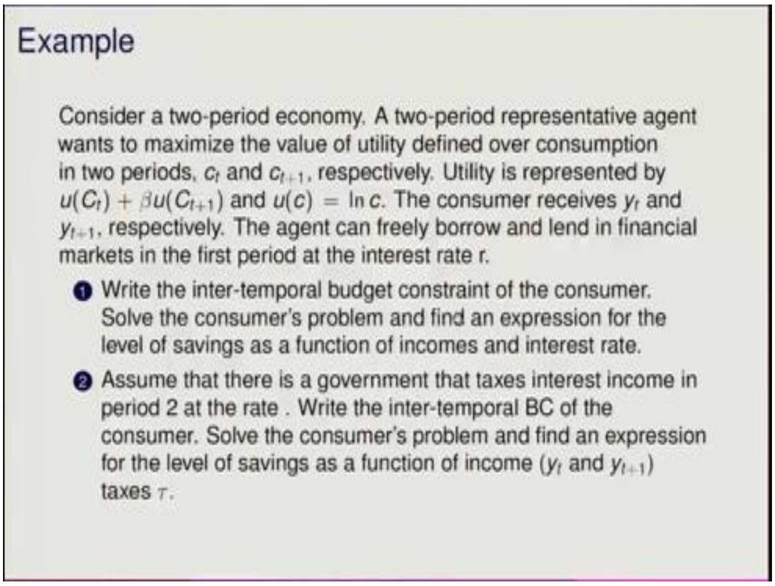


Now let us have one example, I think this will clear your understanding about this model. So, to have the feel about this model, we are going to work out this particular example and then we will have further scenarios to deal with and that will further strengthen your understanding of this topic. So here suppose we have the 2-period economy model 2-period representative agent wants to maximize the value of the utility defined over consumption in 2-period.

So, he has $c_t$ and $c_{t+1}$. Utility is represented by $u(c_t) + \beta u(c_{t+1})$ and $u(c) = ln c$. The consumer receives a $y_t$ and $y_{t+1}$ respectively, the agent can freely borrow and lend in financial market at the rate of interest r. So, this is what we have the $r$. So, we do not have any kind of frictions in the financial market, no asymmetry at all. The rate of interest is same means whatever amount of money that I am going to keep in the bank it is offering a 4% rate of interest. If I am going to borrow from the bank, it is still offering me the same rate of interest. So, there is symmetry in the credit market, there is no asymmetry. Normally the case is when you save money in the bank, your bank charges different rates of interest then then you have the lower lending rates when you lend to the bank, you are being offered lower rate.

But when you borrow from the bank then they will give you at a higher rate. So that we are not considering. That creates market imperfections, we will be examining separately. There they will have the role of limited commitment and then further the good borrower and bad borrowers the great market asymmetry will try to solve. So that part is different. Here for the time being for the sake of simplicity.

We are assuming that the $r$ is same for borrowing and lending. Now, the first question is writing the inter–temporal budget constraint of the consumer, solve the consumer's problem and find an explanation for the level of savings as a function of incomes and interest rate. Assume that there is a government that taxes interest income in period 2 at the rate. Write the inter–temporal budget constraint.

So, let us first solve this part, write the inter–temporal budget constraint of the consumer, solve the consumer's problem and find an explanation for the level of savings as a function of incomes and the interest rate. So, this is how we are going to solve about it.

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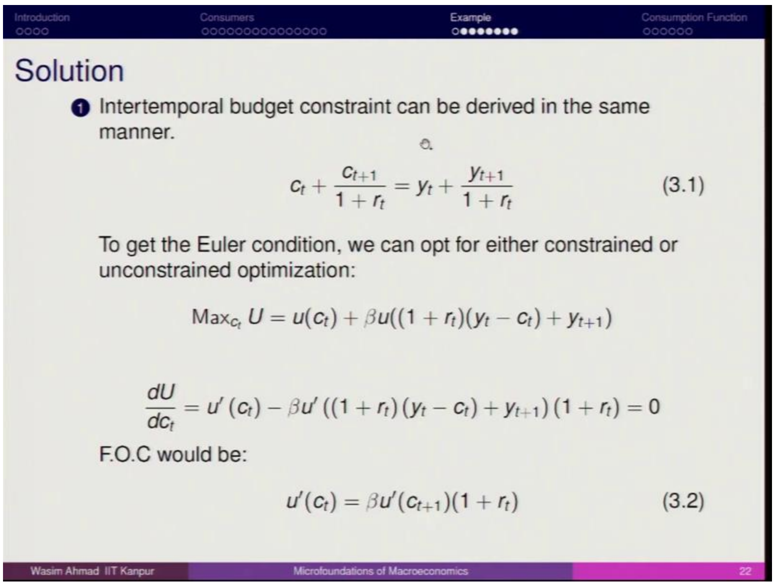


So, if you try to solve then find the value of $s_t$ from future budget constraint and substitute it in current period budget constraint. You are going to get this expression.

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t + \frac{y_{t+1}}{1+r_t}$$

To get the Euler condition what we try to solve through constrained or unconstrained optimization.

$$\max_{c_t} U = u(c_t) + \beta u((1+r_t)(y_t - c_t) + y_{t+1})$$

$$\frac{dU}{dC_t} = u'^{(C_t)} - \beta u'((1+r_t)(Y_t - C_t) + Y_{t+1})(1+r_t) = 0$$

So, this part becomes the $c_{t+1}$ and then you can solve for this. So, you get the Euler conditions.

$$u'(C_t) = \beta u'(1+r_t)(C_{t+1})$$

Euler condition is what here you have the first marginal utility of current consumption is equivalent to the marginal utility of future consumption multiplied by $\beta$ times $1 + r$. So, this is how we get it.

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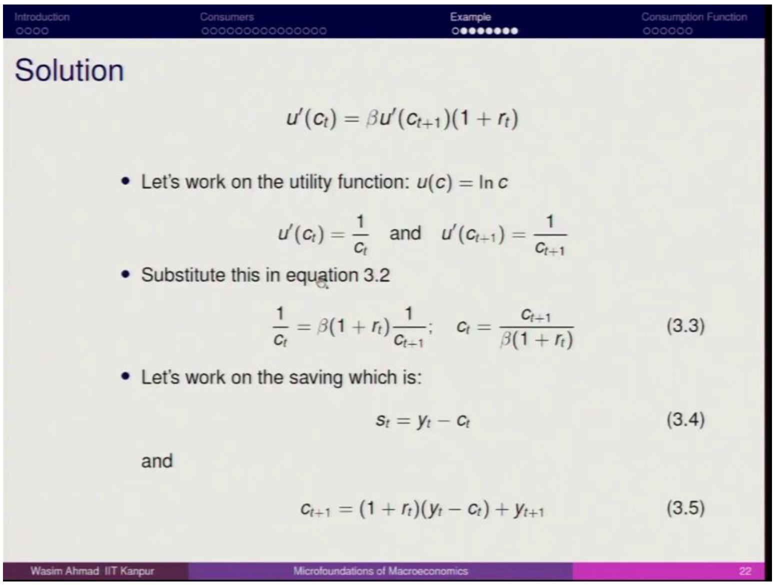


Now, once I have this then what is the next part? Solve the consumer's problem and find an expression for the level of savings as a function of incomes and the interest rate. So here, we have to now, find the level expression for savings. So, how do you find that? So, saving expressions we have got this expression now, let us work it out. So here it becomes. So now, if you think about since I have assumed here, $u(c) = lnc$, so, Euler condition will be what? So, if I am going to get this expression,

$$u'(c_t) = \frac{1}{c_t} \text{ and } u'(c_{t+1}) = \frac{1}{c_{t+1}}$$

Now, substituting this in Euler equation, what is that?

$$\frac{1}{c_t} = \beta\ (1+r_t)\ \frac{1}{c_{t+1}}\text{; or } c_{t+1} = \beta\ (1+r_t)\ c_t$$

So, this is what we are going to solve. So, if we just solve for $c_t$ here, this is what we get the expression.

$$c_t = \frac{c_{t+1}}{\beta\,(1+r_t)}$$

Saving is $s_t = y_t - c_t$. So, I have the $c_t$ expression here I just substituted. So, $s_t = y_t - c_t$. So, I just substitute it here.

$$s_t = y_t - \frac{c_{t+1}}{\beta\,(1+r_t)}$$

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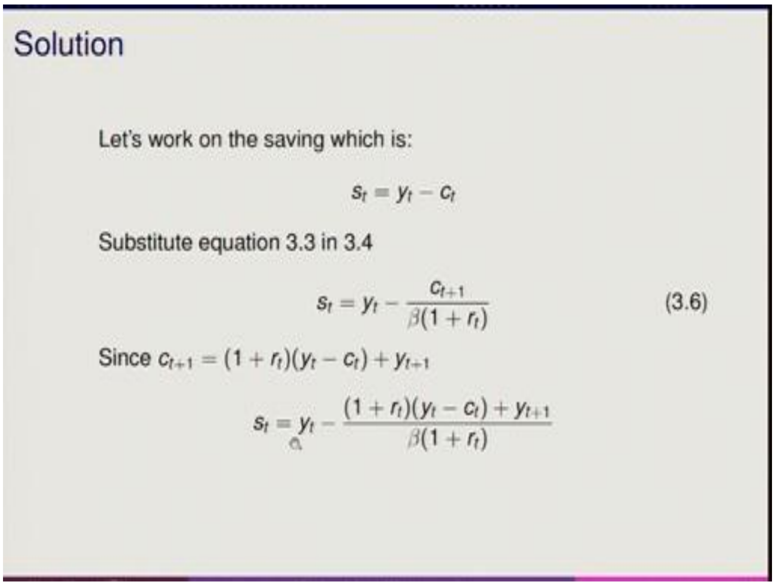


So, you get this expression. So,

$$c_{t+1} = (1+r_t)(y_t - c_t) + y_{t+1}$$

$$s_t = y_t - \frac{(1+r_t)(y_t - c_t) + y_{t+1}}{\beta\,(1+r_t)}$$

So, if you just $c_{t+1}$, if you try to replace this by this expression because we you want to solve the income and interest rate and in the current period only this particular expression has to be. So, we are going to replace this by this.

$$s_t = y_t - \frac{(1+r_t)(s_t) + y_{t+1}}{\beta\,(1+r_t)}$$

So, this is how we try to get it. So, finally we have this expression, if you solve this further, you will have you arrive at this conclusion. So, this is how the saving function can be derived.

$$s_t = \frac{\beta y_t - y_{t+1}/(1+r_t)}{1+\beta}$$

Now, further is the second part. Second part was that let us assume that there is a government that taxes interest income in period 2. So, once I am talking about interest income then I am talking about the proportional tax in period 2.

Write the inter–temporal budget constraint of the consumer solve the consumer's problem and find a solution for the level of saving. So, how it works? It works here. So,

$$c_t + \frac{c_{t+1}}{1 + r_t(1-\tau)} = y_t + \frac{y_{t+1}}{1 + r_t(1-\tau)}$$

So, if you solve for $c_{t+1}$. This is what we have

$$c_{t+1} = (1 + r_t(1-\tau)(y_t - c_t) + y_{t+1}$$

So, if you substitute this here, so, here you must always keep in mind that when I am talking about the interest income. So, here it means that the tax is now, imposed on the interest. So, here it is proportional tax $1-\tau$. So, this is what we get

$$\max_{c_t} U = u(c_t) + \beta u((1 + r_t(1-\tau))(y_t - c_t) + y_{t+1})$$

The first order condition is,

$$\frac{dU}{dc_t} = u'^{(c_t)} - \beta u'((1 + r_t(1-\tau))(y_t - c_t) + y_{t+1})(1 + r_t(1-\tau) = 0$$

Finally, in your Euler equation, what you are trying to get is this part.

$$u'(c_t) = \beta u'(c_{t+1})\,(1 + r_t(1-\tau))$$

So, this is what is the change that you see from the previous Euler condition is this $1-\tau$ and this becomes an important topic. So, here what we see is that if we have the incidence of tax then incidence of tax goes on the when we are talking about the proportional tax which means that some portion some percentage of income will be taxed.

So, here in this case, since we are having the interest income tax, so, here it becomes $(1 + r_t(1-\tau))$. This particular part, it is important that you should understand these dynamics. So, we will continue in the next session from here. Thank you. Thank you so much.