Introduction to Econometrics
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Lecture 55
Relaxing the Assumptions of CLRM-Autocorrelation and Heteroscedasity Part-6

So, that means, if you now compare the Goldfeld Quandt test and the BPG test then you are actually arriving at same type of conclusion.

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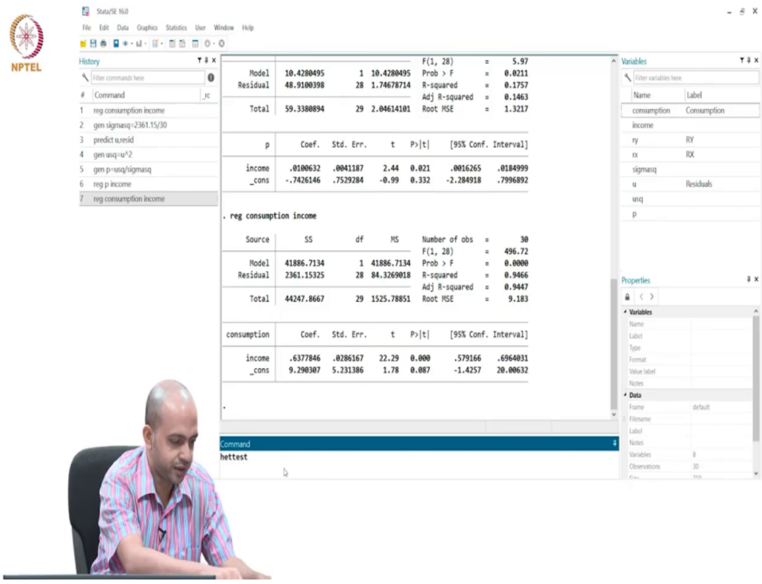

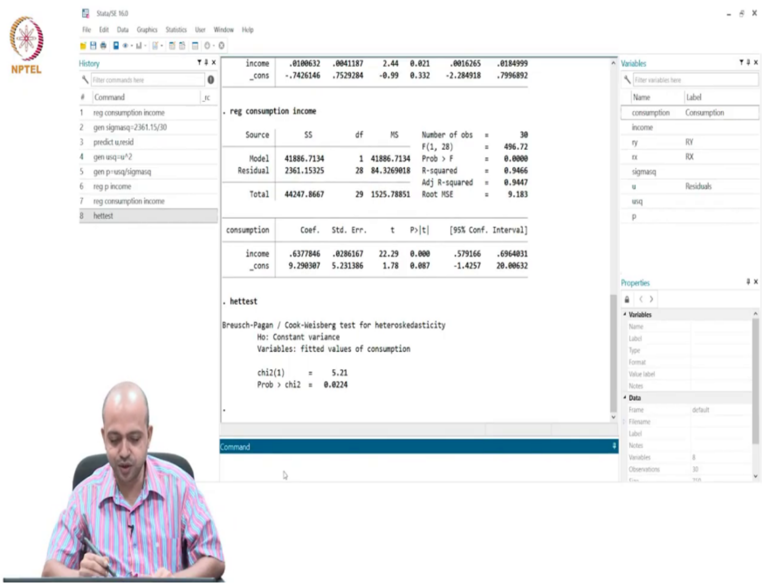

So, in STATA they have a readymade command to conduct this test and that is basically I look, I will explain this, so once you run your original model reg consumption on income then the command for BPG and Goldfeld Quandt test is het test and see the value is 5.21

exactly the same value what we have derived same value, manually whatever we have calculated STATA is also reporting the same chi square value and what is the probability that the calculated value is greater than the tabulated one look at the p value 0.0224, so that means, if you multiply this by 100 you will get 2.24 which is greater than 1 but, less than 5, so that means, it is significant at 5 percent level. So, you can reject your null hypothesis at 5 percent level of significance. So, this is the Briush Pagan and Goldfeld Quandt test.

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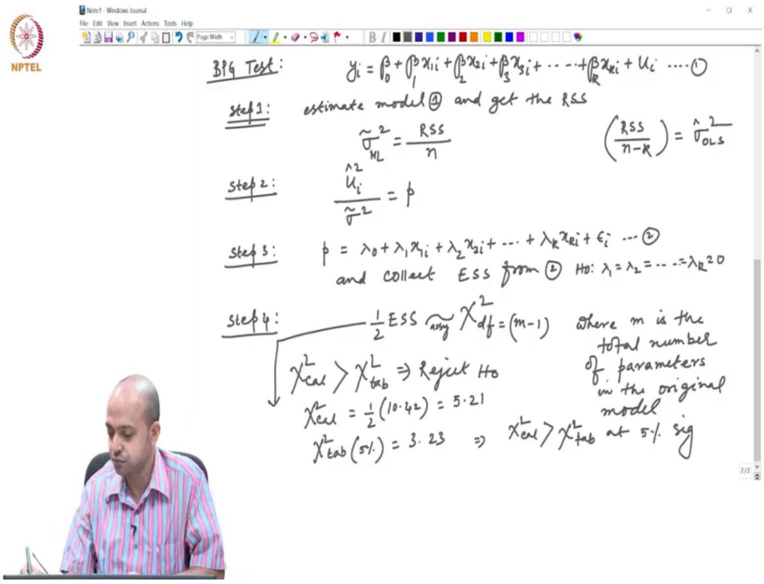

But, there are 2 limitations of this BPG test also. First of all the chi square what I said, this chi square this ESS by 2 is asymptotically follows chi square distribution that means, this particular test statistic follow the chi square distribution only in large sample only in large sample also the sigma square tilde, what we got here to adjust your Ui hat square and that is you see that is the definition of sigma square from the maximum likelihood method and maximum likelihood method is applicable when you have a large sample, but here you have only 30 observations but, these 30 observations may not actually define a large sample. So, that is limitation number 1 half of ESS asymptotically follow chi squared distribution that means, this particular test statistic you can derive only when your sample size is large.

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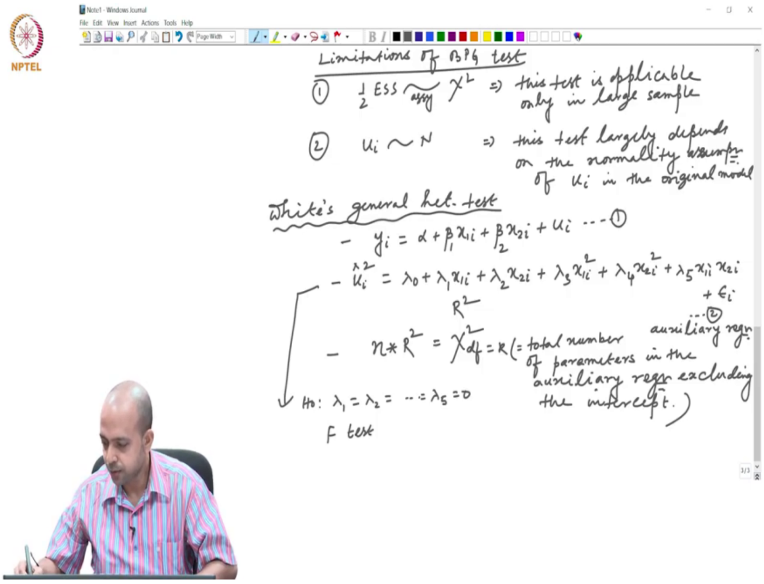

So, limitations of BPG test first one is that half of ESS follows asymptotically chi square distribution that means, this implies this test is applicable only in large sample. second limitation is that in your original model Ui actually follows a normal distribution so these test largely depend on the normality assumption of the error term in the original model, so that means this implies this test largely depends on the normality assumption of Ui in the original model.

To overcome these 2 limitations, there is one more test which is called White General Heteroscedasticity test in short I will write het test, how is this test suppose your original model is yi equals to alpha plus beta1 x1i plus beta2 x2i plus Ui this is your original model and here what they say the test is very simple you estimate this model and then, collect Ui hat square and then you regress this Ui hat square on lambda1 x1i plus lambda2 x2i plus lambda 3 x1i square plus lambda4 x2i square plus lambda 5 x1i x2i plus epsilon i and then, get this is step 1 this is step 2 and then from here you get r square and in step 3 if you multiply these R square that will follow again x chi square distribution with the degrees of freedom equals to k this is let us say equation 1, this is equation 2 which is also known as auxiliary regression.

k equals to total number of explanatory variable in or I will say the total number of parameters in the auxiliary regression but excluding the intercept, so that means from the auxiliary regression alternatively, you can check instead of constructing n multiplied by r square, you can also test lambda 1 equals to lambda 2 equals to dot dot dot lambda 5 equals to 0 or not. Let us say this is my null hypothesis. How will you do that by F statistic and if lambda 1 lambda 2 lambda 5 all are 0 then sigma squared equals 2 what?

That means only lambda 0, so if this is not rejected that means, sigma squared equals only lambda 0 that means, there is no presence of Heteroscedasticity but, if H not is rejected from this auxiliary regression, if you conduct F test that means Ui squared Ui hat square actually equals to lambda 0 plus lambda 1 lambda 2 lambda 3, so that means again I can say that variance of Ui is a function of x. So, this is a different type of functional form we have assumed.

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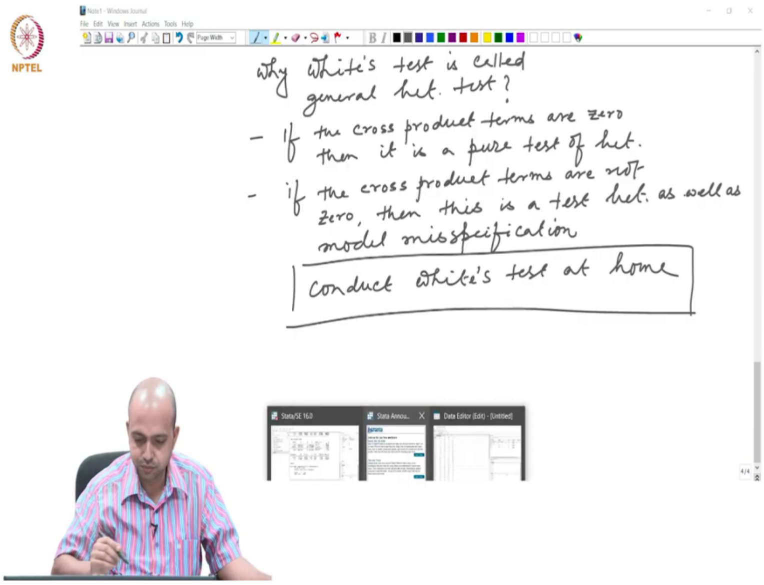

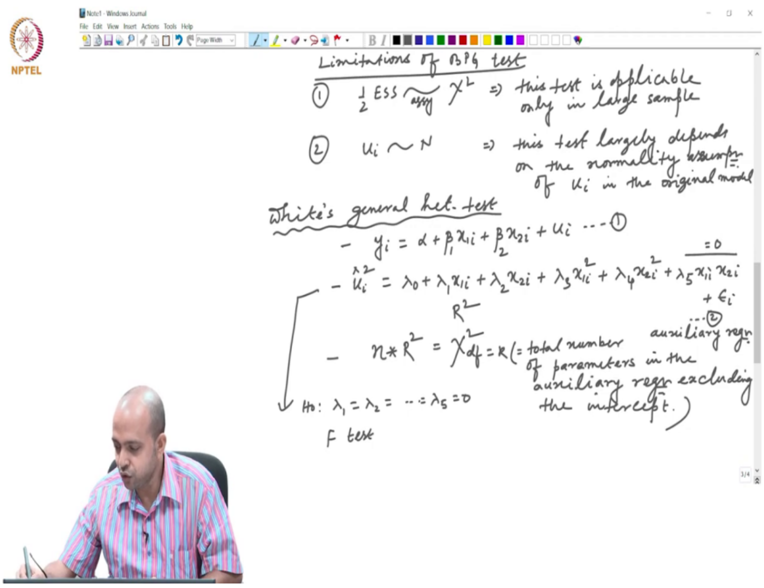

Now, why White's test is called general Heteroscedasticity test can you think of, see the specification of a white test you have Ui hat squared equals to all the explanatory variable it is square and cross product, if this cross product terms are significantly they are 0 if you have more number of explanatory variables you will have more cross product.

So, if the cross-product terms are 0, so that means it is a pure test of Heteroscedasticity. if the cross product terms are not 0 then, this is a test of Heteroscedasticity as well as model misspecification that is why it is called a General test because you are testing Heteroscedasticity as well as model misspecification. Now, you can take the same data set and again you can manually conduct this White's General Heteroscedasticity test and that you take as an assignment, so conduct this test at home conduct.

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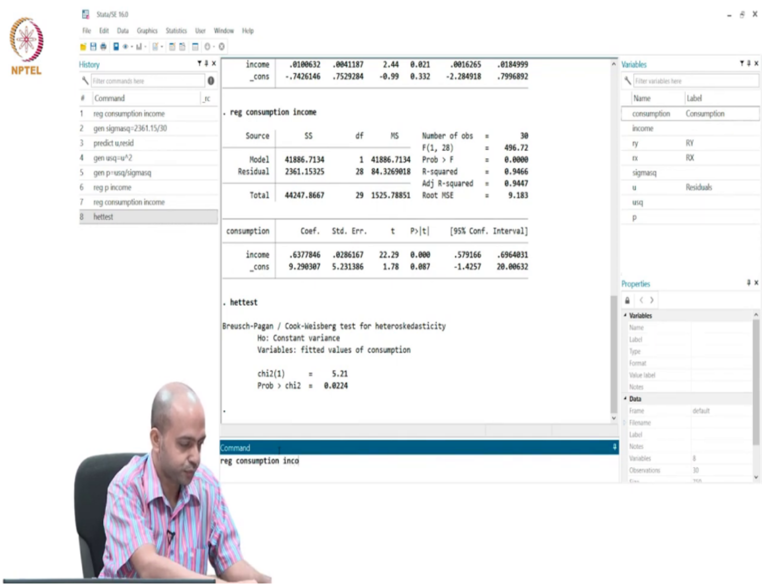

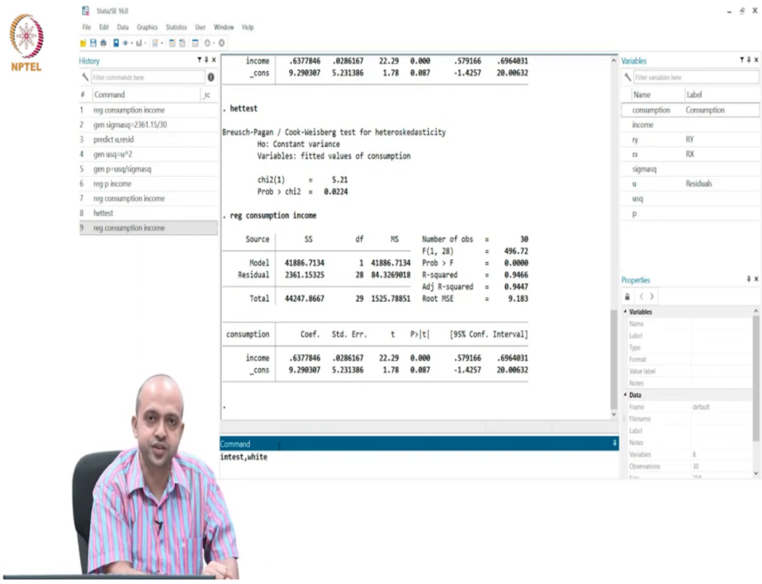

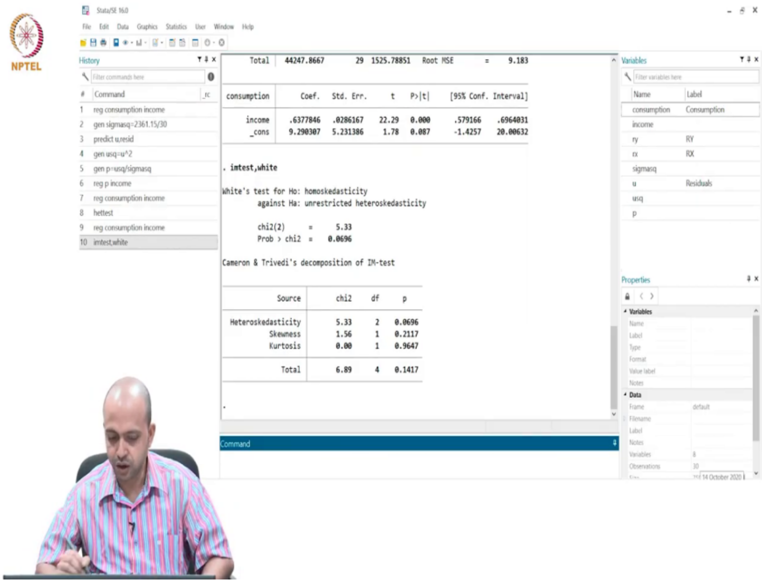

Now, here I will just show you how to do it in STATA. so in STATA once again if you run the original model, reg consumption and then income after that you have to do imtest white, what is the command I am saying imtest white, that is the command for white's general Heteroscedasticity test and from there or you can do or you can see that chi square value is 5.33 and the p value is 0.0696 that means it is significant only a 10 percent level.

But, once again you have to remember that even the White's General Heteroscedasticity test is also a large sample test, which may not be applicable in a small sample in this case, because you have only 30 number of observations, why this is a large sample test?

Because see in your auxiliary regression and Ui hat squared is a function of all your explanatory variable it square and cross product, so with 2 explanatory variable with only 2 explanatory variable you are having 5 parameters, so you can imagine if you have 5 then all the 5 explanatory variable is square and cross product that means 5c2 terms cross product times cross product terms would be there, so many explanatory variable will appear in the auxiliary regression and that will eat up lot of degrees of freedom, so that is why you need to have a large sample to conduct this particular test also.

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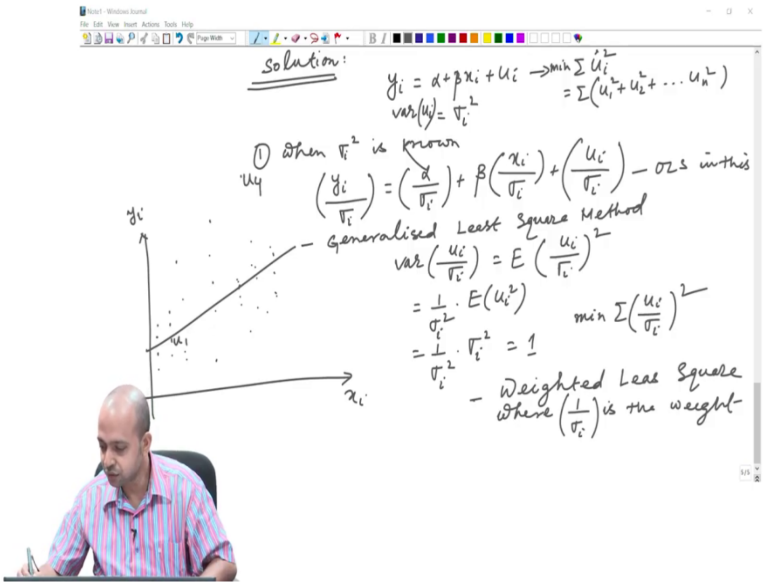

Now, the question is, we have detected Heteroscedasticity but what is the solution and how will you solve. So, the solution is in your model yi equals to alpha plus beta xi plus ui and you are saying that sigma square i sigma square equals to sigma square i so, that means variance of ui in STATA I will write variance of Ui equal to sigma square i, so there are 2 cases when sigma squared i is known to you somehow, you know the error variance sigma square i is known to you, so in that case if you simply divide this equation in STATA of estimating that original equation, you just divide the equation by sigma plus beta xi by sigma i plus Ui by sigma i you divide the equation and then run OLS in the modified regression, if you run OLS in this modified regression, that is called generalised least square method.

Now, the question is with OLS is not applicable in the original model, how come we are able to apply OLS in the transform model, because if you calculate the variance of these Ui by sigma i then that is nothing but expectation of Ui divided by sigma i square, which is nothing but 1 by sigma square i into expectation of Ui squared and what is expectation of Ui square that is also sigma squared i equals to 1 by sigma square i into sigma square i equals to 1. So, that means variance is constant in the modified regression while it was not in the original regression.

Now, if you look at the procedure that means what I am doing in the original regression in this regression while doing OLS we are trying to minimise Ui hat square and here what I am trying to minimise in this generalised model, we are trying to minimise Ui divided by sigma i square, so this method is also known as weighted least square method. Where, 1 by sigma square is basically is the weight, so that means higher the variance, lower is the weight you are attaching to that particular error term.

Now, how it is different from OLS? This is yi in OLS, what was happening, so this was where error term this is my error term suppose, this one is U1 and this is U4, so when you are minimising summation Ui square that means you are basically minimising summation U1 square plus U2 square plus dot dot dot Un square, so that means, you are attaching same weight for all these error terms but, here in the WLS or Weighted Least Square what I am doing am attaching 1 by sigma as the weight.

That means, higher that means, when your distance is too far from the predicted line I will attach only the lower weight, so weight is inversely proportional to the distance of the error term from this so that means, higher the value of the error term lower the weight I am attaching to that particular error term and that should be realist that should be justified also because, this error term contributes less towards constructing this regression line, why should I attach too much weight to this, why should I attach similar weight to U4 with that of U1, U1, because this is too close to the predicted line.

So, that means this is contributing more, this is more important than this error term if you put equal weight then U4 will dominate the summation Ui hat square that is not actually happened that is the problem of OLS that is why in WLS I am saying am attaching weight and weight is actually inversely proportional to the value of this. Higher the U lower would be the weight attached to it that is the logic but, the question is this is all right, you can transform the equation when sigma i square is known. But, when it is unknown, they not will do?

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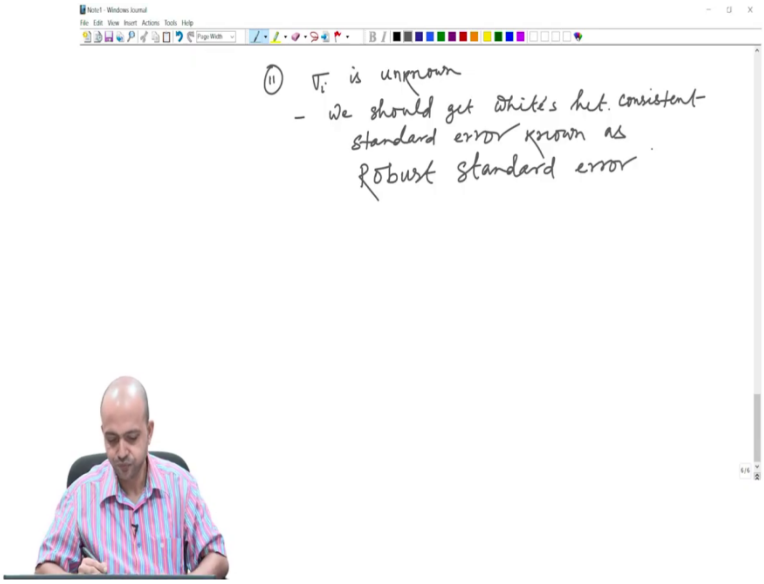

When sigma i is unknown we should get White's Heteroscedasticity consistent standard error actually because, ultimately, your standard error will get affected in presence of Heteroscedasticity, so you have to get this and this particular standard error White Heteroscedasticity consistent standard error that is known as Robust standard error.

And nowadays in all these statistical software's they routinely compute the Robust standard error which is basically White Heteroscedasticity consistent standard error that means assuming there is a presence of heteroscedasticity what is the standard error in large difference between the OLS standard error and this Robust standard error indicates that your data is basically suffering from Heteroscedasticity problem and if you go to your data set once again.

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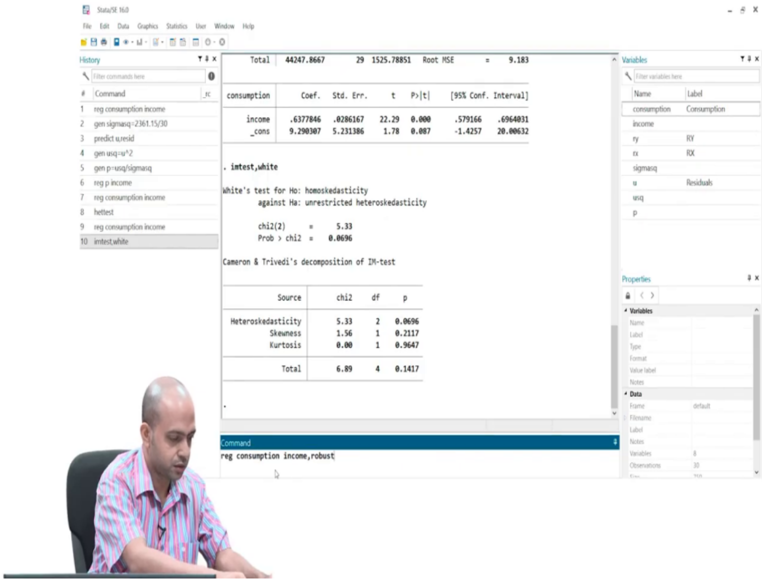

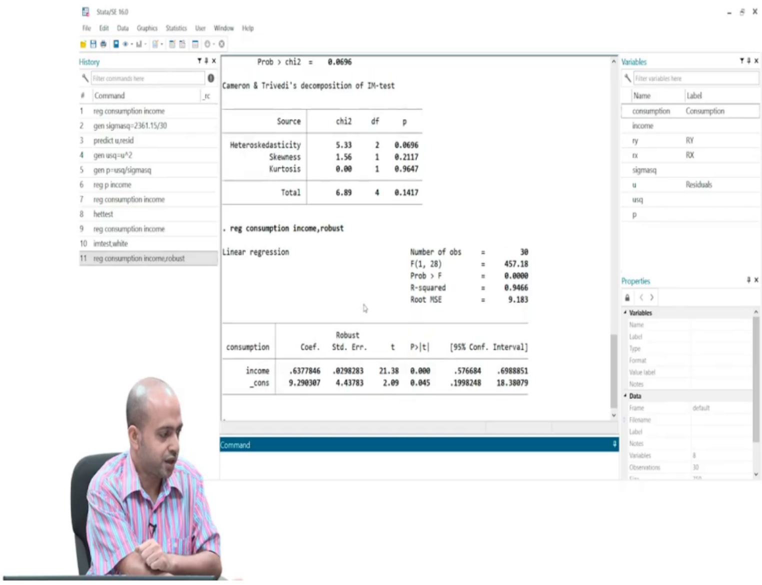

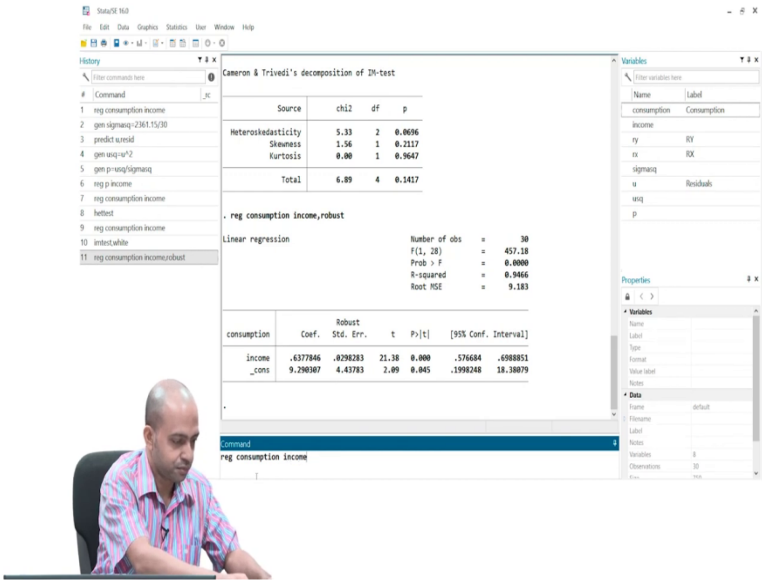

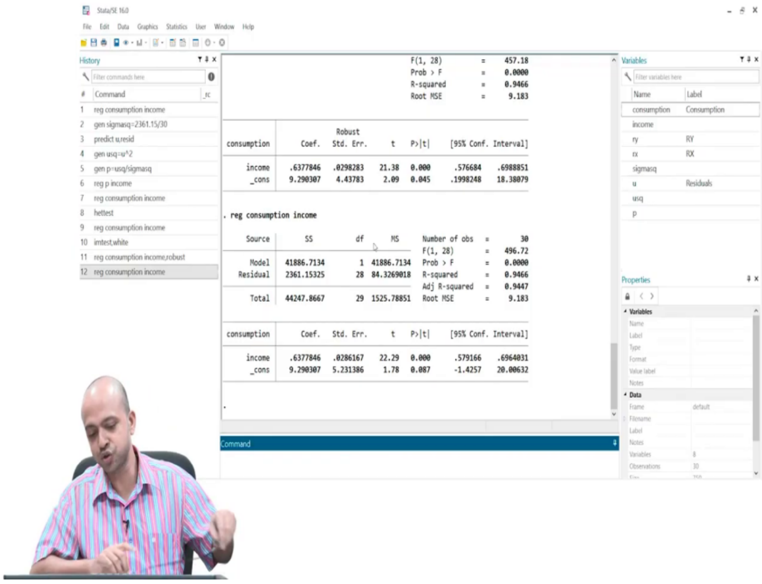

So, if you run this model reg consumption then income after that you have to put a Robust command to get a Robust standard error. Now, you see the standard error for the income coefficient is 0.0298 and in your original model, if you run simply reg consumption on income then your standard error is basically 0.0286 so there is not much of a difference in the standard error that means, the presence of Heteroscedasticity is not so severe and that was prominent from the White Heteroscedasticity test also that you can reject the null only at 10 percent level, this is how you can get a Robust standard error to solve the Heteroscedasticity problem, if at all anything is there. So, with this we are basically closing our discussion on heteroscedasticity.

Now, you see, we have discussed autocorrelation we have discussed multicollinearity and now, we have discussed Heteroscedasticity that means, we have relaxed only 3 assumptions out of 10 we mentioned in the context of classical linear regression model, that means, we assume other 7 assumptions are maintained out of those 7 assumptions, there is 1 more assumption which is also very important known as.

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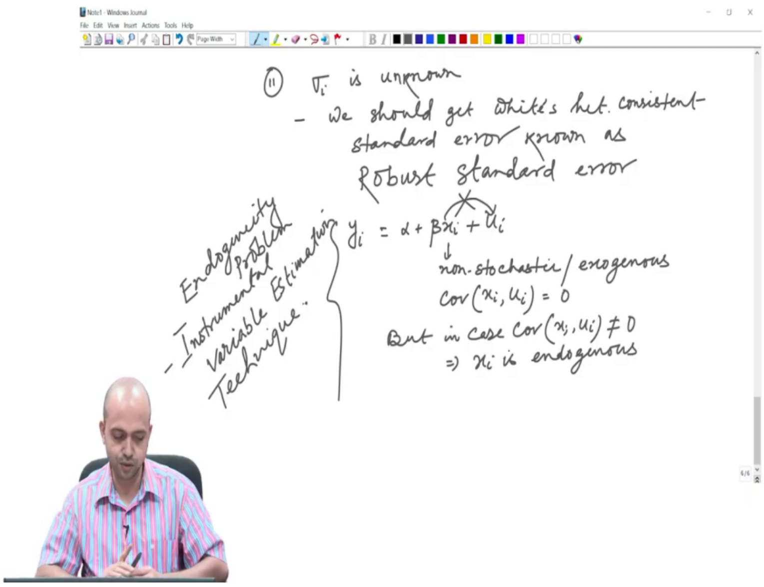

Let us have y equals to alpha plus beta xi plus ui where we assume this xi is basically non stochastic or exogenous that means, covariance of these xi and Ui is actually 0, this is not correlated with this error term but in case covariance between xi and Ui is not equal to 0, then that is called xi is endogenous and it will lead to endogeneity problem.

So, this endogeneity problem if you have in your data set, then your OLS method is not applicable, you have to use instrumental variable estimation technique but, this IV technique or instrumental variable estimation technique is beyond the scope of this basic econometrics the course what we are dealing with that is that comes under the next course, which is applied econometrics.

But, you should aware of this problem also, this is also one of the assumption that we maintained in the context of classical linear regression model that all our explanatory variables, they are exogenous in nature and that means, they are not correlated with the error term. So, with this we are closing our discussion of this module that means relaxing the assumption of classical linear regression model Autocorrelation, Multicollinearity and

Heteroscedasticity detection of the problem consequences and the solution we had discussed. Thank you.