Introduction to Econometrics

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Lecture No. 06
Desirable Properties of the Estimates of the Population Parameters Part – 2

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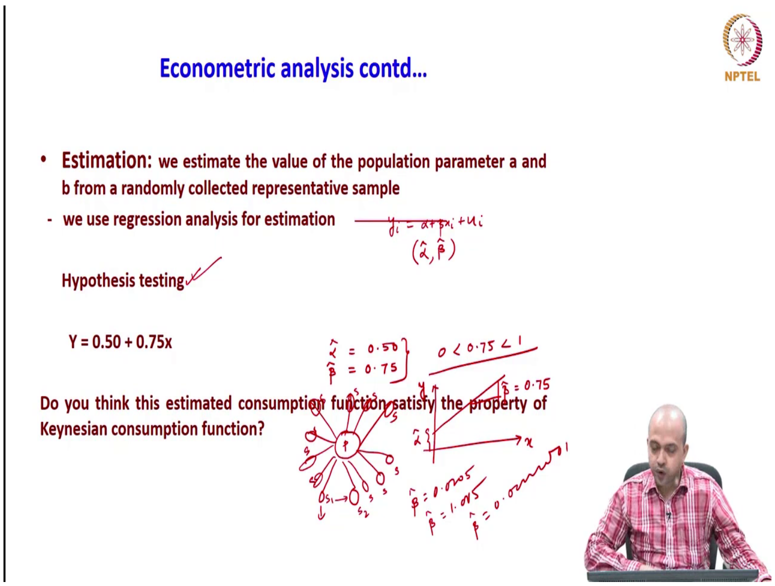

We will discuss the detailed procedure of hypothesis testing in detail because hypothesis testing is the most important part of our econometric analysis. Hypothesis testing is the most important part because if you recall, our objective of econometric analysis was drawing inference about the population parameter by checking whether the coefficient of income is statistically significant or not. If the coefficient is statistically significant, then we can say that income has a significant impact on consumption. That is the inference what we are going to make about our population.

But that is not possible after estimation. For that we need to go for hypothesis testing. That is why we say that econometric analysis is mainly divided into 2 stages- one is estimation and another one is called hypothesis testing or inference making. So, the hypothesis testing is the most important part for this inference making and that is the reason we need to discuss in detail about hypothesis testing and that we will discuss in detail in a later part of our discussion.

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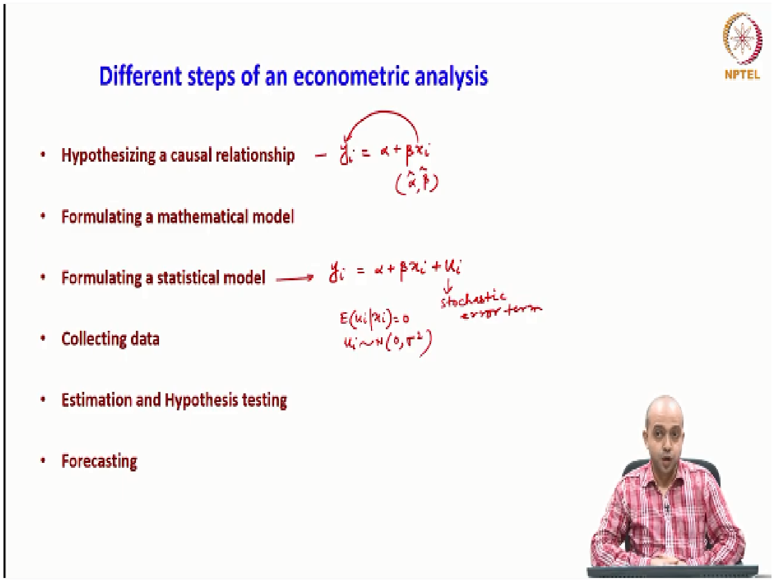

Now, once you go for hypothesis testing then, if you recall what is our next step? It is forecasting or instead of forecasting I would say prediction since we are going to use the cross sectional data. That means with hypothesis testing you have established that my beta hat is significant not only mathematically but also statistically because I have now ensured that even if I collect another sample, my $\hat{\beta}$ value will be significantly different from 0 and it is not by a chance factor. So, that is why you can now use this $\hat{\alpha}$ and $\hat{\beta}$ value for prediction. That means now you can use this model to predict what is going to be somebody's consumption for a given level of income or you can also predict what amount of income is required to sustain a specified value of consumption and these estimates then now can be used for policy making.

But before we use this $\hat{\alpha}$ and $\hat{\beta}$ for policy purpose or prediction or forecasting purpose, econometricians say that you must ensure that your $\hat{\alpha}$ and $\hat{\beta}$ - the estimates of your population parameter- exhibit 3 desirable properties before we actually use them for policy purpose forecasting or prediction. For example, let us say that you have invented a vaccine for the ongoing Coronavirus but before we apply that medicine on the human being you need to examine whether this vaccine is reliable and whether this vaccine is having any side impact or not. Likewise, we must ensure that our estimates of $\hat{\alpha}$ and $\hat{\beta}$ are reliable and they exhibit certain desirable properties before we actually apply for policy purpose. Unless they exhibit those desirable properties, it would be dangerous to use this $\hat{\alpha}$ and $\hat{\beta}$ for policy purpose, like applying the Coronavirus medicine on a human being without testing their desirable properties.

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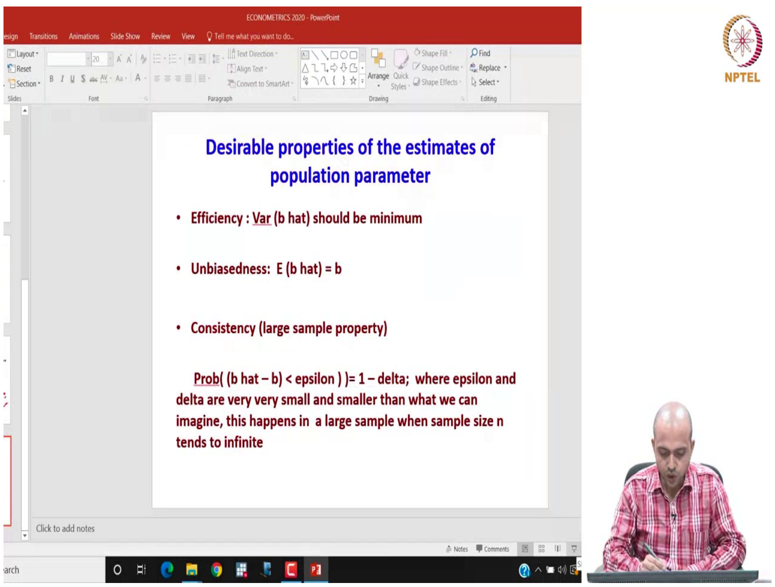

And what are those desirable properties of your $\hat{\alpha}$ and $\hat{\beta}$? There are basically 3 properties that your $\hat{\alpha}$ and $\hat{\beta}$ must exhibit. What are those? First one is called efficiency or efficiency property. Efficiency is defined as the minimum variance of your $\hat{\beta}$.

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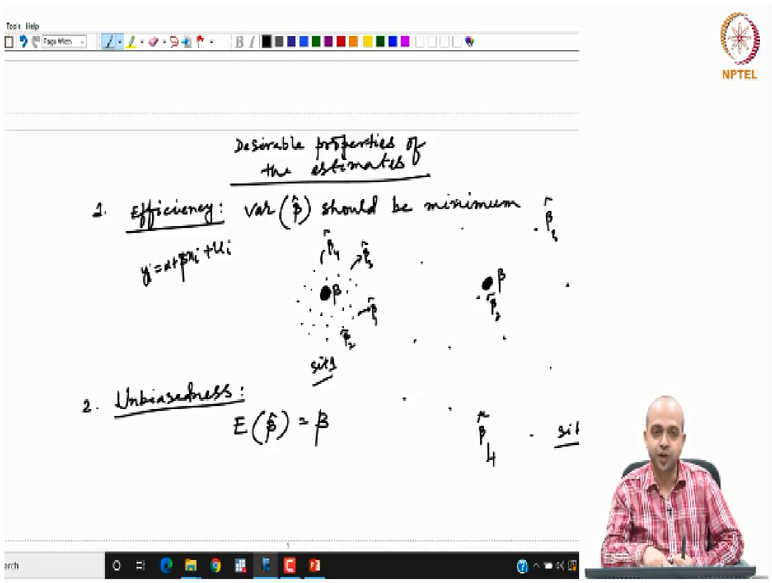

Now, I will explain the efficiency property which is a desirable property of the estimate. First one is called efficiency, which basically says that variance of $\hat{\beta}$ should be minimum. Now I will give you one example that will make your understanding of efficiency very clear. Let us say that this is your true population parameter which is β and I am giving an example, say we play a game, let us say this is a point what we are targeting with a gun, so that means we have to hit this point and if you shoot and say you are making 100 shooting, then in all your 100 shooting what will happen?

Sometimes you will hit exactly here in the β, sometimes you will hit here, you will hit there like this. And let us say that all these points are $\hat{\beta}$. Let us say this is $\hat{\beta}_1$, this is $\hat{\beta}_2$, $\hat{\beta}_3$, $\hat{\beta}_4$ like that these are all points. So that means what this minimum variance property say is that in all your shootings the variation or the deviation from the actual point should be minimum. Then only we can say that these estimates of true population parameter $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ are actually reliable.

But suppose, your point is here and you are targeting here, that means you have a huge variance and I cannot rely because today you are hitting here from this sample and next if

I collect another sample that $\hat{\beta}$ is here. So that means when my objective is to draw inference about the true population parameter with the sample statistic, my sample statistic should lie as close as possible to the true population parameter β. Then only we can say that there is preciseness in our inference. Otherwise if your variance of the $\hat{\beta}$ is too large, that means you are basically hitting like this, this is your β and your $\hat{\beta}_1$ is here, $\hat{\beta}_2$ is here, $\hat{\beta}_3$ is so this is situation 1.

And let us say this is situation 2. So I will say that minimum variance property will be satisfied here in this context. That means when I am collecting several samples from the population, I will get a variance of he $\hat{\beta}$ and that variance basically should be minimum. That is the desirable property. Why this is required? Because when the variance is minimum, there would be preciseness in our decision making or in our inference making. When the variance is much wider or much larger, then there would not be any preciseness in the inference.

That is why minimum variance property says that variance of $\hat{\beta}$ should be minimum. But the way we have explained this case, it appears like we are going to draw 100 samples from β, from the population and we will estimate 100 such he $\hat{\beta}$ values and then we will compute that variance and then we will check whether their efficiency property is satisfied or not. But actually, in reality we will collect only one sample and we will estimate only one he $\hat{\beta}$ but we will ensure the minimum variance property. Why? How? This is because of a statistical procedure. If you recall, we have assumed that $Y_i = \alpha + \beta X_i + u_i$ and we have assumed something about this u_i that is u_i follows a normal distribution. When u_i follows a normal distribution that will ensure that he $\hat{\beta}$ will also follow a normal distribution and that is the variance of the distribution I am talking about.

So, you do not have to collect 100 samples but you will collect only one sample and estimate only one he $\hat{\beta}$ but we will ensure the minimum variance property. We will check the minimum variance property of that he $\hat{\beta}$ -whether it is satisfied or not from one sample only because of the distributional assumption of this u_i.

And the minimum variance property would be satisfied only when we estimate our model under certain assumptions and those assumptions basically explain an idealistic situation. Those assumptions will be discussed later. For the time being you just keep in mind that there are certain assumptions that we must make to describe an idealistic situation under which when we estimate the model and our $\hat{\beta}$ value will show the minimum variance property. That is called efficiency.

Then the second desirable property that we need is unbiasedness. This basically says that expectation of $\hat{\beta}$ equals to β. That means when you collect several samples from the population there would be fluctuation in your $\hat{\beta}$ value because of the sampling fluctuation. But if you take mean of all those $\hat{\beta}$ values, that would be equal to your true population parameter β and that is what is called unbiasedness property.

That means average of your entire he $\hat{\beta}$ is β and once again I am not going to collect several samples. Rather I will collect only one sample but because of that statistical property and because of those assumptions that we make, that will ensure whether expectation of $\hat{\beta}$ equals to β or not. So, we must ensure that this property is also satisfied before we actually use this he $\hat{\beta}$ for policy purpose or inference making. That is called unbiasedness.

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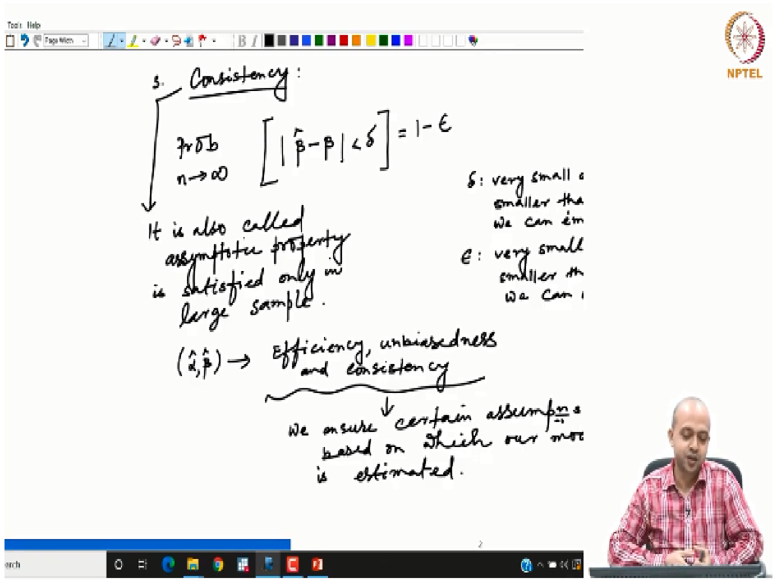

And then the third property is called consistency. Consistency property is a little involved. I will explain it step by step. Let us say that you have estimated your $\hat{\beta}$ value as this and your true population parameter is this, so the difference between these should be minimum. That means it should be less than delta, where delta is a very, very small quantity, smaller than whatever you can actually imagine.

So, that means $\hat{\beta}$ is very, very close to the true population parameter β and what is the probability of getting such $\hat{\beta}$? Probability of getting such $\hat{\beta}$ =1-ε, where ε again is a very, very small quantity, smaller than what you can actually imagine, when your sample size tends to infinity. So, this is the consistency property. So if you think step by step, you can easily understand that your $\hat{\beta}$ value is the estimated value of the true population parameter β.

So, the difference between $\hat{\beta}$ and β should be minimum. That is why I said the difference is actually lower than delta where delta is very small and smaller than what we can imagine. Similarly, epsilon is also very small, very small and smaller than what we can imagine. So, that means probability of getting such beta is almost 1 and when will that happen? When sample size tends to infinity.

While you can ensure unbiasedness and efficiency in a small sample also, consistency property or also called the asymptotic property, is satisfied only in large sample. So, it is called asymptotic property which is satisfied only in large sample. So that means our $\hat{\beta}$ would satisfy efficiency, unbiasedness and consistency and when will our $\hat{\alpha}$ and $\hat{\beta}$ satisfy these 3 properties? When we ensure certain assumptions based on which our model is estimated. So, we have to make certain assumptions and those assumptions will actually ensure that your $\hat{\alpha}$ and $\hat{\beta}$ will satisfy efficiency, consistency and unbiasedness. But in reality when we are collecting the data and estimating the model, it is quite likely that either one or more than one of those assumptions are violated.

And when the assumptions are violated, then either one or more of the desirable properties of your estimates will also be violated and your $\hat{\alpha}$ and $\hat{\beta}$ will not satisfy efficiency, unbiasedness and consistency when the assumptions of that idealistic situations are violated. That is why when we estimate the model, we have to specifically check whether any of those assumptions are violated or not. If violated, then we have to apply appropriate remedial measures. Those things we will discuss in a later part of our discussion.

So, right now what we have discussed is these are the properties they must satisfy and must exhibit before we actually apply this $\hat{\alpha}$ and $\hat{\beta}$ for policy purpose.

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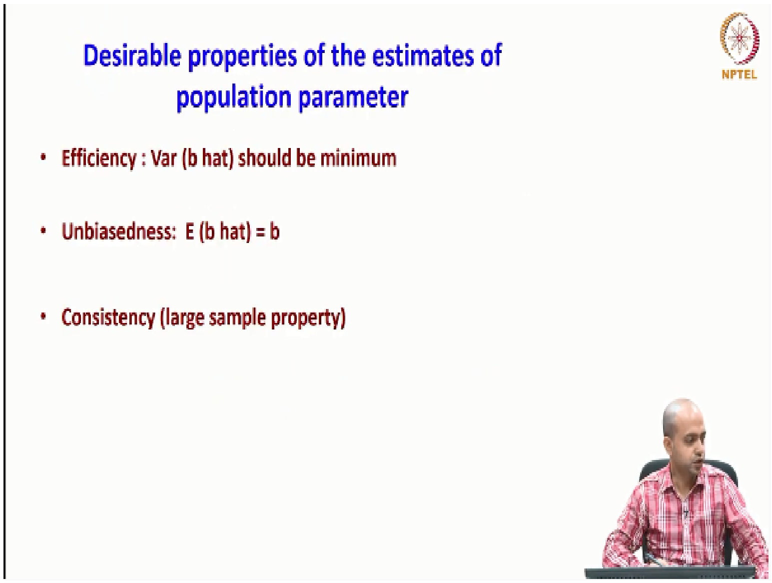

That is what is mentioned here, alright. So these are the desirable properties. So, with this we are closing our discussion of this module. Basically in the last two lectures what we have discussed is the introductory part of econometrics- the definition, different steps and the 3 desirable properties that our estimates must exhibit before we use them for policy purpose. We will discuss in our next class those assumptions and the actual procedure of estimation and hypothesis testing. Thank you.