Environmental & Resource Economics
Professor Sabuj Kumar Mandal
Department of Humanities and Social Sciences
Indian Institute of Technology Madras
Optimum extraction of renewable resources and Tragedy of Commons Part - 1

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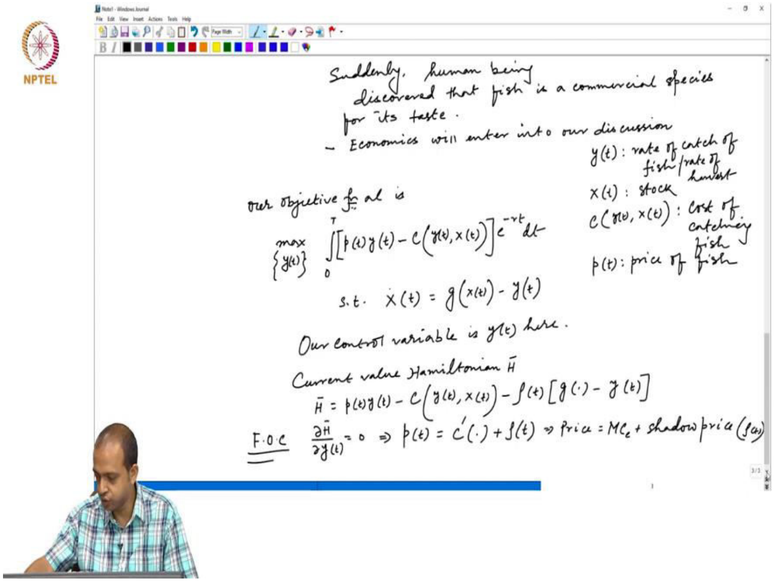

Now what is happening suddenly, let us assume, suddenly human being discovered that that fish is a commercial species for its test. So, obviously, economics will come in, economics will enter into our discussion. So, once again, let us assume that yt is rate of extraction or rate of catch of fish xt indicate stock, then c is again a function of cost of catch and then pt is the price of fish.

So, our objective functional is maximize. What we are going to maximize? Integrations 0 to t again, pt into yt the revenue Pt into yt minus c of yt into xt, and e to the power minus rt, dt. The same objective functional in the like non-renewable resource, but the constraint will change as x dot t, that means, rate of change of the stock is equals to g of xt growth minus harvest. So, earlier it was x dot t minus yt.

If you go back and see in the context of non-renewable resource, since non-renewable resource cannot grow within a economically feasible time horizon, we denoted x dot t equals to minus of yt. Here x dot equals to g of xt minus yt. So, our control variable once again is yt. So, our control variable is yt here. Rate of catch or you can say that rate of harvest, that is it rate of harvest.

Now, the optimization requires once again the first step is what do you remember? The first step is formulating the current value Hamiltonian. So, I will take it in the next page or let us say that the current value Hamiltonian denoted by H bar, H bar equals two what I will write, simply pt into yt minus c function yt into xt t minus rho t into this g of xt, g function minus yt. This the current value Hamiltonian, and maximization not optimization required differentiating this function with respect to yt.

Differentiating this function with respect to yt, and say dt equals to 0 that is coming from the first order condition. And these implies, if I differentiate the current value of Hamiltonian with respect to date of extraction what I will get pt equals to C prime plus rho t. So, that means, price should be equal to cost of extraction plus the shadow price which is similar to our case previous case. So, price would be equals to marginal cost of extraction plus shadow price, which is rho t. So, this condition is similar to our non-renewable resource also.

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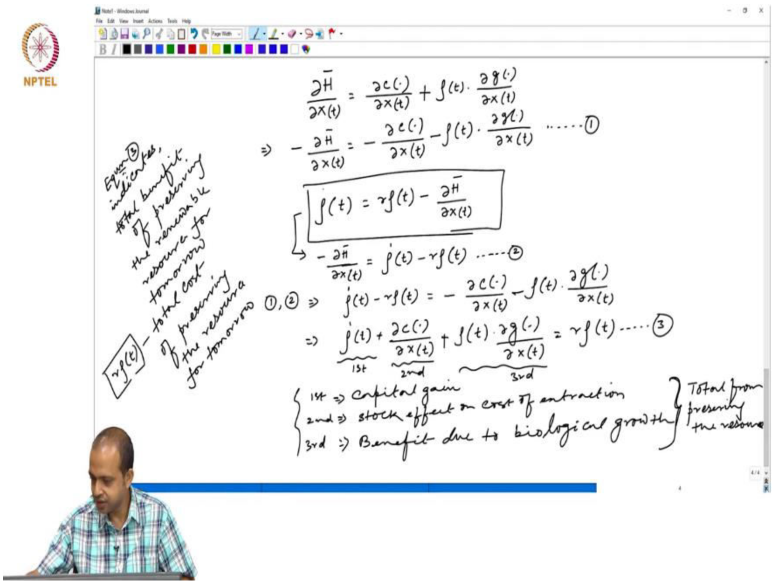

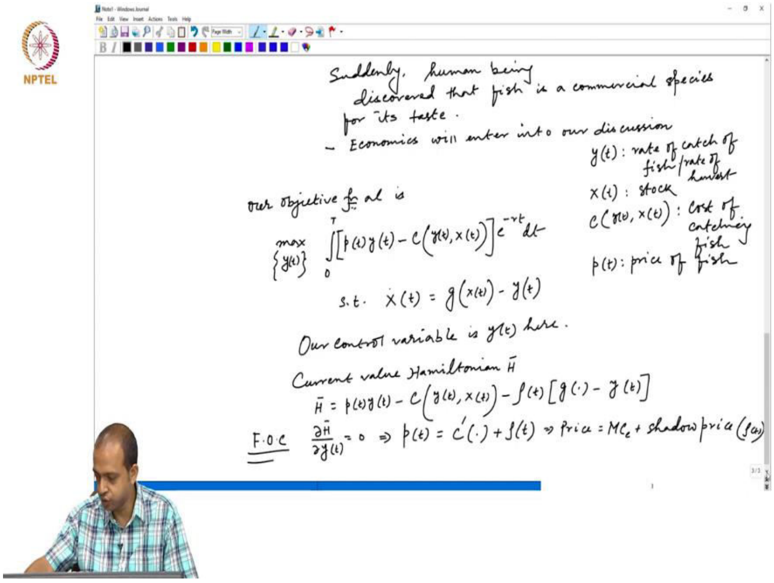

Now, from the second step what we need to do is basically differentiating the Hamiltonian with respect to xt and set it equals to 0. Sorry, this is you know second order condition is coming from this actually that second order condition from second order condition we need to differentiate this function with respect to delta xt.

So, this is basically how I am deriving the second order necessary condition, and this is equals to if we differentiate H function with respect to xt, then what I will get, x is here on the cost function, so I will get delta c delta xt and I have x in the growth function also minus rho t into delta g delta xt. So, from here what I can say that minus of this equals to delta xt equals to minus of delta c, delta xt minus or sorry plus of rho t into del g del xt.

Now, what we will do this minus del H bar del xt that is basically equals to we will put the value of this from our earlier condition. What is that condition we derived? If you recall from earlier condition that we get that roh t this is let us say equation 1. This is let us say equation one what we have derived so far, and then, what we will do, previously we got one condition rho dot t if you recall rho dot t equals to r into rho t minus r into rho t minus del H bar del xt this is one condition, which we derived earlier this condition.

So, that means, I can write del H bar del xt equals to rho t minus of this. So, from here what I can write that minus del H bar del xt equals to I can say rho t minus r into rho t. So, what I will do, I will put this value here. Let us say this is equation 2. So, from equation 2, 1 and 2 or I can write that rho dot t minus r into rho t equals to minus del c del xt plus rho t into del g del xt or what we can write that, what we can write is that, this r into rho t we will take other side and then we will say that rho dot t plus del c dot del xt minus, so what we will get, minus rho t into del g into del xt.

So, this is this is let us say, we are calling equation 3, sorry, this this then would become equals to r into t, this is let us say equation 3. Now, we need to interpret this equation 3. So, let me check whether we did everything correctly this is this so, this should be equals to minus of this so, rho dot t minus, rho dot t equals to r into rho t minus del H equals to this. So, sorry, this is plus actually.

This is plus, so this would become minus, this has become minus. And then, if this is minus then rho dot t equals to this so, ultimately, this would become this so ultimately this would become this should become plus. Now, in this equation 3 basically again like the non-renewable resource case, in this case equation 3 indicates total benefit of preserving the renewable resource. So, equation 3 indicates total benefit of preserving the resource, renewable resource for tomorrow.

That is what, alright, this is what we see. So, here this should be plus. So, unlike, the non-renewable resource when we talk about renewable resource the current value Hamiltonian is this plus lambda t into this because if you put it plus here, that means, you are deducting the extraction plus adding the growth. So, this should be plus, and as a result of which when you differentiate this, this should be also plus.

So, there are three components in equation 3. Let us say this is the first component, first, then this is the second component and this is the third component. So, first component indicate capital gain like the non-renewable resource, capital gain. Second component indicates what? Similar to the non-renewable resource stock effect on cost of extraction on marginal cost of extraction or on cost of extraction. Now, the third component of the total benefit is actually missing in non-renewable resource.

This additional term, what is this indicating look at this. This is basically benefit for biological growth, biological growth was not there in the context of non-renewable resource. If I preserve the resource for tomorrow, tomorrow my fishes will grow, and if I want to convert that growth into benefit obviously, I have to multiply that with roh t because that is the shadow price, that is why third component basically indicate benefit due to biological growth, which was absent earlier in the context of non-renewable resource.

So, these three are the benefit, if I preserved the resource for tomorrow, and that should be equals to at least r into rho t. I can always catch the fish, and the revenue part I can keep it in bank, will give interest at the rate of r, so my total benefit would be r into rho t. That is what we meant. So, this is called, this is the total benefit. So, this is total benefit from preserving the resource and r into rho t that basically indicates total cost of preserving the resource for tomorrow.

So, at equilibrium, total benefits should be equals to total cost, that is why this equation says, total benefit of preserving one unit of renewable resource should be equals to total cost or preserving the resource at equilibrium. So, when these two things are equal then we say that we are on the optimum path of extraction.

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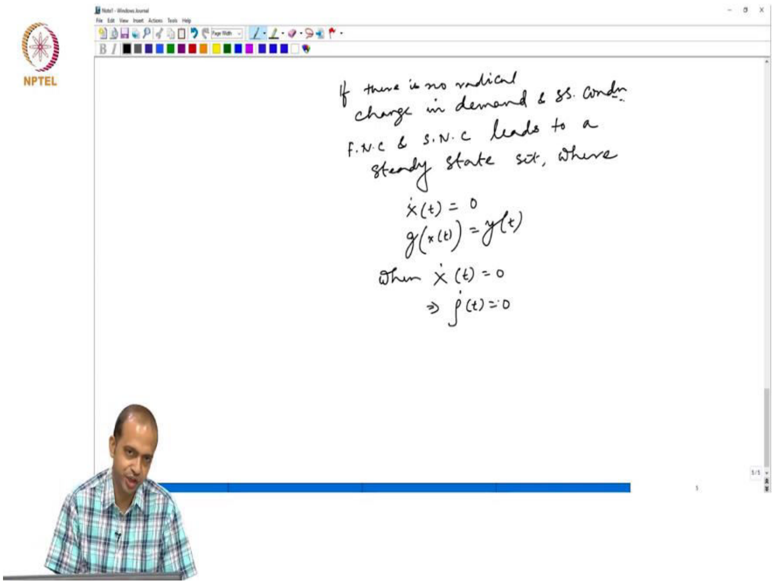

So, at steady state, so if there is no radical change in demand and cost condition, there is no radical change in demand and supply condition then first order necessary condition FNC and SNC second order necessary condition leads to a steady state situation where basically x dot t growth of the stock equals to 0 that means g of xt growth is basically equals to the rate of harvest.

And when x dot t equals to 0 that implies there is no capital gain also rho dot t also equals to 0 because this capital gain arises due to change in stock only. If there is no change in stock, then there is no change in price. Because we assume there is no radical change in demand and supply condition. So, with this we are closing our discussion today. and we will meet again in our next class to discuss the remaining portion of this resource economics. Thank you.